FYUG Even Semester Exam., 2025

MATHEMATICS

(2nd Semester)

Course No. : MATDSC–152

(Integral Calculus and Vectors)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) Express

$$\int_a^b f(x)\,dx$$

as the limit of a sum.

(b) Evaluate :

$$\int \frac{x^2+x-1}{x^3+x^2-6x}\,dx$$

(c) Show that

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

2. Answer any *one* of the following :

(a) (i) Write down the geometrical interpretation of

$$\int_a^b f(x)\,dx$$

(ii) State and prove the fundamental theorem of integral calculus. 1+5

(b) (i) Show that

$$\lim_{n\to\infty}\left\{\frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{2n}\right\}=\log_e 2$$

(ii) Show that

$$\int_0^{\pi/2}\log(\sin x)\,dx=\frac{\pi}{2}\log\left(\frac{1}{2}\right)$$

UNIT—II

3. Answer any *two* of the following : 2×2=

(a) Evaluate :

$$\int_0^{\pi/2}\sin^5 x\,dx$$

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(b) Show that

$$\int \tan^n x\,dx=\frac{\tan^{n-1}x}{n-1}-\int \tan^{n-2}x\,dx$$

where $n\in\mathbb{N}$.

(c) Find $\int \sec^3 x\,dx$.

4. Answer any *one* of the following : 10

(a) Obtain a reduction formula for

$$\int_0^{\pi/2}\sin^m x\cos^n x\,dx$$

where $m, n\in\mathbb{N}\cup\{0\}$.

(b) (i) Evaluate

$$\int_0^{\pi/2}\sin^n x\,dx$$

and use it to find the value of the integral

$$\int_0^1\frac{x^n}{\sqrt{1-x^2}}\,dx$$

where $n\in\mathbb{N}\cup\{0\}$. 4+2=6

(ii) Let

$$I_{m,n}=\int_0^{\pi/2}\cos^m x\sin nx\,dx$$

where $m, n\in\mathbb{N}\cup\{0\}$.

Show that

$$I_{m,n}=\frac{1}{m+n}+\frac{m}{m+n}I_{m-1,n-1}$$ 4

(Turn Over)

UNIT—III

5. Answer any *two* of the following : 2×...

(a) Write down the formula to find the length of a curve in Cartesian and polar form.

(b) Write the formula for finding the volume of the solid of revolution formed by rotating $y = f(x)$ about x-axis between $x = a$ and $x = b$.

(c) Find the area of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$

6. Answer any *one* of the following :

(a) (i) Find the area above the x-axis, included between the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 2ax$.

(ii) Find the surface area of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about its base.

(b) (i) Find the length of the arc of the parabola $y^2 = 8x$ measured from the vertex to one extremity of the latus rectum. 5

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(ii) Find the volume and area of the curved surface of a paraboloid of revolution formed by revolving the parabola $y^2 = 4x$ about the x-axis, and bounded by the section $x = a$. 5

UNIT—IV

7. Answer any *two* of the following : 2×2=4

(a) Show that three non-zero vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if and only if $[\vec{a}, \vec{b}, \vec{c}] = 0$.

(b) Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$$

(c) If $\vec{a} = \hat{i} - 2\hat{j} - 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, and $\vec{c} = \hat{i} + 3\hat{j} - 2\hat{k}$, then find $|\vec{a} \times (\vec{b} \times \vec{c})|$.

8. Answer any *one* of the following : 10

(a) (i) Prove that

$$[\vec{b} \times \vec{c}, \vec{c} \times \vec{a}, \vec{a} \times \vec{b}] = [\vec{a}, \vec{b}, \vec{c}]^2$$ 5

(ii) Find the vector equation of a plane passing through a given point and perpendicular to a given vector. 5

(Turn Over)

(b) (i) If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three non-coplan[ar] vectors, then prove that any vect[or] $\vec{r}$ can be expressed as

$$\vec{r} = \frac{[\vec{b}, \vec{c}, \vec{r}]}{[\vec{a}, \vec{b}, \vec{c}]}\vec{a} + \frac{[\vec{c}, \vec{a}, \vec{r}]}{[\vec{a}, \vec{b}, \vec{c}]}\vec{b} + \frac{[\vec{a}, \vec{b}, \vec{r}]}{[\vec{a}, \vec{b}, \vec{c}]}\vec{c}$$

(ii) Find the vector equation of a sphere.

UNIT—V

9. Answer any *two* of the following :

(a) Show that

$$\frac{d}{dt}(\vec{u} + \vec{v}) = \frac{d\vec{u}}{dt} + \frac{d\vec{v}}{dt}$$

(b) Write down the geometrical interpretation of $\frac{d\vec{r}}{dt}$.

(c) Find $\frac{d}{dt}(\vec{a} \times \vec{b})$, where $\vec{a} = t^2\hat{i} - 3t\hat{k}$, $\vec{b} = -4t^3\hat{j} + t\hat{k}$.

10. Answer any *one* of the following :

(a) (i) Let f be a vector function defined by $f(x, y, z) = 3x^2y - y^3z^2$. Find $\vec{\nabla} f$ at $(1, -2, 1)$.

(ii) If $\vec{v}$ is a constant vector, show that $\text{div}\,\vec{v} = \vec{0} = \text{curl}\,\vec{v}$. 2+2=4

(iii) Find $\frac{d\vec{r}}{dt}$, where

$$\vec{r} = e^{-5t}(3\cos t\hat{i} + 2\sin t\hat{j})$$ 3

(b) (i) If
$$\vec{a} = \sin\theta\hat{i} + \cos\theta\hat{j} + \theta\hat{k}$$
$$\vec{b} = \cos\theta\hat{i} - \sin\theta\hat{j} - 3\hat{k}$$
$$\vec{c} = 2\hat{i} + 3\hat{j} - 3\hat{k}$$

then find $\frac{d}{d\theta}(\vec{a} \times (\vec{b} \times \vec{c}))$ at $\theta = \frac{\pi}{2}$. 5

(ii) Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. Prove the following :

(1) $\vec{\nabla} f(r) = f'(r)\,\vec{\nabla} r$

(2) $\vec{\nabla} r = \frac{1}{r}\vec{r}$

(3) $\vec{\nabla}\frac{1}{r} = -\frac{1}{r^3}\vec{r}$ 2+1+2=5

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