FYUG Even Semester Exam., 2025

MATHEMATICS

(2nd Semester)

Course No. : MATDSC–151

(Analytical Geometry)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following questions : 2×2=4

(a) Find the transformed equation of the line $x\cos\alpha + y\sin\alpha = p$, when the axes are rotated through an angle α.

(b) Prove that the equation

$$6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$$

represents a pair of perpendicular lines.

(c) The origin is shifted to the point $(3, -1)$ and the axes are then rotated through an angle $\tan^{-1}\left(\frac{3}{4}\right)$. Find the coordinates of the point $(5, -2)$ in the new coordinate system.

2. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Prove that a homogeneous second degree equation

$$ax^2 + 2hxy + by^2 = 0$$

always represents a pair of straight lines passing through the origin.

(b) Prove that the two pairs of straight lines

$$ax^2 + 2hxy + by^2 = 0 \text{ and}$$
$$a^2x^2 + 2h(a+b)xy + b^2y^2 = 0$$

have the same bisectors.

(c) Prove that the straight lines represented by the equation

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

will be equidistant from the origin if $f^4 - g^4 = c(bf^2 - ag^2)$.

(d) If, by the orthogonal transformation without change of origin, the expression $ax^2 + 2hxy + by^2 + 2gx + 2fy + c$ be changed to

$$a'x'^2 + 2h'x'y' + b'y'^2 + 2g'x' + 2f'y' + c'$$

then show that—

(i) $a'b' - h'^2 = ab - h^2$

(ii) $g'^2 + f'^2 = g^2 + f^2$ 2+2=4

UNIT—II

3. Answer any *two* from the following questions : 2×2=4

(a) Find the value of k for which the circles $x^2 + y^2 + 5x + 3y + 7 = 0$ and $x^2 + y^2 - 8x + 6y + k = 0$ are orthogonal.

(b) Find the radical centre of the circles

$$x^2 + y^2 + 4x + 7 = 0,$$
$$2x^2 + 2y^2 + 3x + 3y + 9 = 0 \text{ and}$$
$$2x^2 + 2y^2 + y = 0$$

(c) Find the equation of the circle through the points of intersection of the circles $x^2 + y^2 + 2x + 3y - 7 = 0$ and $x^2 + y^2 + 3x - 2y - 5 = 0$ and through the point $(1, 2)$.

4. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Find the equation of the circle orthogonal to both the circles $x^2+y^2+3x-5y+6=0$ and $4x^2+4y^2-28x+29=0$ and whose centre lies on the line $3x+4y+1=0$.

(b) Prove that two tangents can be drawn from a point to a parabola and if these two tangents be perpendicular to each other, the locus of their point of intersection is the directrix.

(c) Find the condition that the line $lx+my=n$ is a tangent to the ellipse $ax^2+by^2=1$.

(d) If sy and $s'y'$ be the perpendiculars from the foci upon any tangent to an ellipse, then prove that $sy \cdot s'y' = b^2$, where $2b$ is the length of minor axis of the ellipse.

UNIT—III

5. Answer any *two* from the following questions : 2×2

(a) Find the pole of the line $lx+my=n$ with respect to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(b) Show that if the polar of a point P with respect to a conic passes through another point Q, then the polar of Q passes through P.

(c) If the tangent at any point P of a conic meets the directrix in K, then show that the angle KSP is a right angle, S being the corresponding focus.

6. Answer *either* (a) and (b) *or* (c) and (d) : 10

(a) Prove that the sum of the reciprocals of two perpendicular focal chords of a conic is constant. 5

(b) If the polar of a point (α, β) with respect to the parabola $y^2=4ax$ touches the circle $x^2+y^2=a^2$, then prove that the point (α, β) lies on the hyperbola $4x^2-y^2=4a^2$. 5

(c) Find the equation of the chord of the conic $\frac{l}{r}=1+e\cos\theta$, joining the two points on the conic, whose vectorial angles are $\alpha+\beta$ and $\alpha-\beta$. 5

(d) Show that the locus of the poles of chords of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which subtend a right angle at the centre is

$$\frac{x^2}{a^4}+\frac{y^2}{b^4}=\frac{1}{a^2}-\frac{1}{b^2}$$ 5

UNIT—IV

7. Answer any *two* from the following questions : 2×...

(a) Define shortest distance between two skew lines. What is the shortest distance between two intersecting lines?

(b) Find the equation of the sphere through the circle $x^2+y^2+z^2=25$, $x+2y-z+2=0$ and the point $(1, 1, 0)$.

(c) Find the equation of the sphere described on the join of $(2, -3, 4)$ and $(-1, 0, 5)$ as diameter.

8. Answer *either* (a) and (b) *or* (c) and (d) :

(a) Show that the equation to the plane containing the straight line $\frac{y}{b}+\frac{z}{c}=1$, $x=0$ and parallel to the straight line $\frac{x}{a}-\frac{z}{c}=1$, $y=0$ is $\frac{x}{a}-\frac{y}{b}-\frac{z}{c}+1=0$ and if $2d$ be the shortest distance between the lines, then show that $d^{-2}=a^{-2}+b^{-2}+c^{-2}$.

(b) Find the equation of the sphere which passes through the origin and touches the sphere $x^2+y^2+z^2=56$ at the point $(2, -4, 6)$.

(Continued)

(c) A plane passes through a fixed point (α, β, γ) and cuts the coordinate axes in *A*, *B* and *C*. Prove that the locus of the centre of the sphere *OABC* is given by 5

$$\frac{\alpha}{x}+\frac{\beta}{y}+\frac{\gamma}{z}=2$$

(d) Find the length and equation of the shortest distance between the lines 5

$$\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5} \text{ and }$$
$$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$$

UNIT—V

9. Answer any *two* from the following questions : 2×2=4

(a) Find the equation of the cone, whose vertex is origin and the guiding curve is given by $x+2y+3z=4$ and $5x^2+7y^2-3z+2=0$.

(b) Prove that the direction cosines of a generator of a cone whose vertex is origin satisfy the equation of the cone.

(c) What do you mean by guiding curve and generator of a cylinder?

(Turn Over)

10. Answer *either* (a) and (b) *or* (c) and (d) :

(a) A variable plane is parallel to the plan $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ and meets the axes in A, and C respectively. Prove that the circl ABC lies on the cone

$$\left(\frac{b}{c}+\frac{c}{b}\right)yz+\left(\frac{c}{a}+\frac{a}{c}\right)zx+\left(\frac{a}{b}+\frac{b}{a}\right)xy=0$$

(b) Find the equation of the cylinder generated by the lines parallel to the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$, the guiding curve being the conic $z=2,\ 3x^2+4xy+5y^2=1$.

(c) Find the equation of the right circular cone with vertex at the point (3, 2, 1), semi-vertical angle 30° and the axis having direction ratios 1, 4 and 3.

(d) Find the equation of the right circular cylinder of radius 3 whose axis passes through the point (2, 1, 0) and has direction ratios –1, 2 and 3.

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