

2023/FYUG/ODD/SEM/
STADSC-102T/074

FYUG Odd Semester Exam., 2023
(Held in 2024)

STATISTICS

(1st Semester)

Course No. : STADSC-102T

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks
for the questions

SECTION—A

Answer ten questions, selecting two from each

Unit :

$2 \times 10 = 20$

UNIT—I

1. Define function and continuity of a function at a point.
2. State Leibniz's rule for successive differentiation.

3. Show that

$$\lim_{x \rightarrow 0} \frac{x \cdot e^{1/x}}{e^{1/x} + 1} = 0$$

UNIT—II

4. Define Jacobian.
5. Define points of inflection of a function. Write the criteria to find such points.
6. If $f(x, y)$ is a function of x and y , then under what condition $f(x, y)$ is neither maximum nor minimum?

UNIT—III

7. Evaluate $\int \frac{dx}{2\sqrt{x}}$.
8. Evaluate $\int_0^{\infty} x \cdot e^{-x} dx$.
9. Define beta and gamma functions.

UNIT—IV

10. Define order and degree of a differential equation with example.

11. Find the differential equation of the family of curves $y = e^{mx}$, where m is an arbitrary constant.

12. Define exact differential equation.

UNIT—V

13. Define partial differential equation with example.
14. Define order and degree of partial differential equation with example.
15. Eliminate arbitrary constants a and b from $Z = (x - a)^2 + (y - b)^2$ to form the partial differential equation.

SECTION—B

Answer five questions, selecting one from each unit : 10×5=50

UNIT—I

16. (a) Prove that a differentiable function is always continuous. 3

(4)

(b) If $y = \sin^{-1} x$, then show that
 $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$

(c) Find
 $\lim_{x \rightarrow 0} \frac{x \cdot e^x - \log(1+x)}{x^2}$

17. (a) State and prove Euler's theorem for homogeneous function.

(b) Test the continuity and differentiability of the function

$$f(x) = |x - a|$$

at $x = a$.

(c) If $u = \log \frac{x^2 + y^2}{xy}$, then prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$$

UNIT—II

18. (a) Find the maximum and minimum value of the function

$$f(x) = 2x^3 - 21x^2 + 36x - 20$$

(5)

(b) If

$$x_1 + x_2 + x_3 = u$$

$$x_2 + x_3 = uv$$

$$x_3 = uvw$$

then find

$$\frac{\partial(x_1, x_2, x_3)}{\partial(u, v, w)}$$

(c) Find the point of inflection of the curve

$$y = \frac{x^3}{a^2 + x^2}$$

(a) Examine if $x^{1/x}$ possess a maximum or minimum.

(b) Show that the function

$$f(x) = x^2 - 2xy + y^2 + x^2 + y^2$$

is minimum at origin.

(c) If

$$y_1 = \cos x_1$$

$$y_2 = \sin x_1 \cos x_2$$

$$y_3 = \sin x_1 \sin x_2 \cos x_3$$

$$y_n = \sin x_1 \sin x_2 \dots \sin x_{n-1} \cos x_n$$

then find the Jacobian of $y_1, y_2, \dots, y_n$ with respect to $x_1, x_2, \dots, x_n$.

UNIT—III

20. (a) Evaluate

$$\iint_E y \, dx \, dy$$

over the part of the plane bounded by the lines $y = x$ and parabola $y = 4x - x^2$.

- (b) Evaluate

$$\iint_R \sqrt{x^2 + y^2} \, dx \, dy$$

where R is the region of XY -plane bounded by the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$.

21. (a) Prove that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.
 (b) Show that $\Gamma(n) = (n-1)!$.
 (c) Prove that

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}} dx = \frac{1}{(a+b)^m \cdot a^n} B(m, n)$$

UNIT—IV

22. (a) Solve :

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$$

Solve :

$$(b) \quad (x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$$

Solve :

$$(a) \quad \frac{dy}{dx} + \frac{y}{(1-x^2)^{3/2}} = \frac{x + (1-x^2)^{1/2}}{(1-x^2)^2}$$

Solve :

$$(b) \quad \frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}$$

UNIT—V

Solve :

$$p + 3q = 5z + \tan(y - 3x)$$

Solve :

$$(b) \quad xyp + y^2q = zxy - 2x^2$$

Solve :

$$(a) \quad \frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y + 37\sin 3x = 0$$

Solve :

$$(b) \quad \frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$$
