2025/FYUG/EVEN/SEM/
MATDSC–252/219

FYUG Even Semester Exam., 2025

MATHEMATICS

(4th Semester)

Course No. : MATDSC–252

(Mechanics—I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following : 2×2=4

(a) State the law of triangle of forces.

(b) Define moment of a force.

(c) State a set of necessary and sufficient conditions for the equilibrium of a system of coplanar forces acting on a rigid body.

2. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) State and prove Lami's theorem. 1+5=6

(b) A heavy uniform rod of length a rests with one end against a smooth vertical wall, the other end being tied to a point of the wall by a string of length l. Prove that the rod may remain in equilibrium at an angle θ to the wall given by

$$\cos^2\theta = \frac{l^2 - a^2}{3a^2}$$ 4

(c) If three forces acting on a rigid body, keep it in equilibrium, prove that either all the forces meet at a point or all of them are parallel to one another. 4

(d) Forces P, Q, R and S act along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$ and $\overrightarrow{DA}$ of a cyclic quadrilateral $ABCD$, taken in order, where A and B are the extremities of a diameter. If they are in equilibrium, then prove that

$$R^2 = P^2 + Q^2 + S^2 + \frac{2PQS}{R}$$ 6

(Continued)

UNIT—II

3. Answer any *two* of the following : 2×2=4

(a) Define coefficient of friction and angle of friction.

(b) Write two laws of limiting friction.

(c) Define CG of a body.

4. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) A uniform ladder is in equilibrium with one end resting on the ground and the other against a vertical wall. If the ground and wall be both rough, the coefficient of friction being μ and μ' respectively and if the ladder be on the point of slipping at both ends, then show that the inclination of the ladder to the horizon is given by

$$\tan\theta = \frac{1 - \mu\mu'}{2\mu}$$ 5

(b) What height can a particle attain, being at rest inside a hollow sphere of radius a if coefficient of friction is $\frac{1}{\sqrt{3}}$? Also prove that the greatest distance from vertical diameter is $\frac{a\mu}{\sqrt{1+\mu^2}}$, where μ is the coefficient of friction. 5

(Turn Over)

(c) Find the centre of gravity of a sector of a circle subtending an angle 2α at the centre. 5

(d) Find the CG of a uniform lamina bounded by a parabola and a double ordinate. 5

UNIT—III

5. Answer any *two* of the following : 2×2=4

(a) Write expressions for radial and cross-radial (transverse) components of both velocity and acceleration.

(b) Define SHM. Write also the expression for time period of SHM.

(c) A point moving in a straight line with SHM has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2. Show that the period of motion is

$$2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

6. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) The acceleration of a point moving in a plane curve is resolved into two components, one is parallel to the initial

line and the other along the radius vector. Prove that these components are

$$-\frac{1}{r\sin\theta}\frac{d}{dt}(r^2\dot{\theta})$$

and

$$\frac{\cot\theta}{r}\frac{d}{dt}(r^2\dot{\theta}) + \ddot{r} - r\dot{\theta}^2$$ 4

(b) Derive expressions for tangential and normal components of acceleration. 6

(c) A particle is moving with SHM of amplitude a and periodic time T. Prove that

$$\int_0^T v^2 dt = \frac{2\pi^2 a^2}{T}$$ 5

(d) The speed v of a particle moving along the axis of X is given by the relation $v^2 = n^2(8ax - x^2 - 12a^2)$. Show that the motion is simple harmonic with centre $x = 4a$ and amplitude $2a$. Find also the time from $x = 4a$ to $x = 6a$. 5

UNIT—IV

7. Answer any *two* of the following : 2×2=4

(a) A particle moves in a straight line under an attraction towards a fixed point on the line, varying as the square of the

distance from the fixed point. Show that the velocity v at a distance x from the fixed point is given by

$$v^2 = 2\mu\left(\frac{a-x}{ax}\right)$$

(b) A particle is projected from a fixed point into vacuum with velocity u in a direction making an angle $\alpha\left(\neq\frac{\pi}{2}\right)$ with the horizontal. Write expressions for horizontal range and the maximum height attained by the particle.

(c) A particle slides down the outside of a smooth vertical circle starting from rest at the highest point. Draw a rough diagram and write the equation of motion.

8. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) A particle moves in a straight line from a distance a towards the centre of force, the force being $\frac{\mu}{x^3}$ at a distance x from the centre. Show that it reaches the centre in time $\frac{a^2}{\sqrt{\mu}}$. 5

(b) A shot fired at a mark in the horizontal plane through the point of projection goes a metre beyond the mark when the

angle of elevation is α. If the angle of elevation is β, it falls b metre short of the mark. Show that the proper elevation to hit the mark is

$$\frac{1}{2}\sin^{-1}\left(\frac{a\sin 2\beta + b\sin 2\alpha}{a+b}\right)$$ 5

(c) Prove that the path of a projectile in vacuum is a parabola. Also show that the equation to the path may be written in the form

$$y = x\tan\alpha\left(1-\frac{x}{R}\right)$$

where R is the horizontal range and α is the angle of projection. 5

(d) A particle is projected up an inclined plane from the foot of the plane with a velocity u at an angle α to the horizon. Find the range on the inclined plane. 5

UNIT—V

9. Answer any *two* of the following : 2×2=4

(a) State the principle of conservation of linear momentum.

(b) Show that, Impulse = Change in momentum.

(c) State Newton's experimental law for *(i)* direct impact and *(ii)* oblique impact.

10. Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

(a) State and prove principle of energy (principle of work). 5

(b) A gun of mass M fires a shell of mass m horizontally and the energy of explosion is such as would be just sufficient to project the shell vertically to a height h. Show that the velocity of recoil of the gun is

$$\left\{\frac{2m^2gh}{M(m+M)}\right\}^{1/2}$$

5

(c) A ball impinges directly upon another ball at rest and itself reduces to rest by the impact. If half of the kinetic energy is destroyed in the collision, find the coefficient of restitution. 5

(d) A particle falls from a height h in time t upon a fixed horizontal plane. It rebounds and reaches the maximum height h' in time t'. Show that *(i)* $t' = et$, $h' = e^2h$ and *(ii)* the whole distance (up and down) described by the particle before it has finished rebounding is

$$\left(\frac{1+e^2}{1-e^2}\right)h$$

5

★★★