

2025/FYUG/EVEN/SEM/
STADSM-252/117

FYUG Even Semester Exam., 2025

STATISTICS

(4th Semester)

Course No. : STADSM-252

(Mathematical Statistics)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

UNIT—I

1. Answer any *two* of the following questions :

2×2=4

- (a) Define discrete and continuous random variables with illustration.
- (b) Define probability mass function (p.m.f.) of a random variable.
- (c) State the properties of cumulative distribution function.

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2. Answer any one of the following questions : 10

(a) (i) Define p.d.f. and cumulative distribution function of a random variable. 2+2=4

(ii) If X be any discrete random variable having p.m.f.

$$P(X = x) = \frac{1}{a^x}, x = 1, 2, 3 \dots$$

then find the value of a . 2

(iii) The diameter of a electric cable, say x is a continuous random variable having p.d.f.

$$f(x) = 6x(1-x), 0 \leq x \leq 1$$

1. Check whether $f(x)$ is a p.d.f.
2. Determine the number b , such that $p(X < b) = p(X > b)$. 4

(b) (i) A random variable X has the p.d.f.

$$f(x) = Cx^2, 0 \leq x \leq 1 \\ = 0, \text{ otherwise}$$

Find—

1. The value of C ;
2. $P(\frac{1}{4} \leq X \leq \frac{3}{4})$. 1+3=4

(3)

(ii) A discrete random variable X has the following probability distribution :

$X = x$:	0	1	2	3	4	5	6
$P(X = x)$:	0	$2k$	$3k$	k	$2k$	k^2	$7k^2$
$X = x$:	7						
$P(X = x)$:	$2k^2 + k$						

Find—

1. the value of k ;
2. $P(X \geq 6)$;
3. $P(X < 6)$;
4. $P(2 < X < 3)$. 1+2+2+1=6

UNIT—II

3. Answer any two of the following questions : 2×2=4

- (a) Define mathematical expectation.
- (b) Prove that $E(X^2) \geq [E(X)]^2$.
- (c) Prove that $E(a + bX) = a + bE(X)$.

4. Answer any one of the following questions : 10

- (a) (i) State and prove the additive property of mathematical expectation. 5

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(ii) A random variable X has the following probability distribution :

$X = x$	0	1	2	3
$P(X = x)$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{24}$	$\frac{1}{8}$

Find $E(X)$, $V(X)$ and $E(2X - 1)^2$.

1+2+2=5

(b) (i) If X and Y be two independent random variables, then prove that—

$$v(aX + bY) = a^2 v(X) + b^2 v(Y) \quad 5$$

(ii) Let n dice are thrown. Find the expectation at the sum and product of the points on them.

$$\text{Prove that } v(a - bx) = b^2 v(Y) \quad 3+2=5$$

UNIT—III

5. Answer any two of the following questions : $2 \times 2 = 4$

- (a) Define moment-generating function.
- (b) Prove that $\phi_X(ct) = \phi_{CX}(t)$, where $\phi_X(t)$ is the characteristic function of X .
- (c) Explain the uniqueness property of m.g.f.

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6. Answer any one of the following questions : 10

- (a) (i) Prove that m.g.f. of the sum of independent random variables is equal to the product of the m.g.f. of those random variables. 3
- (ii) Prove that m.g.f. is not independent of change of origin and scale. 4
- (iii) Prove that

$$\mu'_r = \text{coefficient of } \frac{t^r}{r!} \text{ in } M_X(t),$$

symbols hold their usual meaning. 3

- (b) (i) Define cumulant generating function. Prove that cumulant generating function is not independent of change of origin and scale. $2+4=6$

- (ii) Define characteristic function. State the properties of characteristic function. $2+2=4$

UNIT—IV

7. Answer any two of the following questions : $2 \times 2 = 4$

- (a) Define bivariate random variable.
- (b) Define joint probability mass function.
- (c) Define joint probability distribution function.

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8. Answer any one of the following questions : 10

(a) (i) If the bivariate probability distribution of (X, Y) is

$Y \rightarrow$ $X \downarrow$	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{3}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

then find—

- marginal probability of X ;
- marginal probability of Y ;
- $P(X \leq 1, Y \geq 3)$;
- $P(X = 1 | Y = 3)$. $2+2+2+2=8$

(ii) What is marginal probability distribution? 2

(b) (i) Define conditional distribution. 2

(ii) A bivariate random variable (X, Y) has the joint probability

$$f(x, y) = 8xy, \quad 0 < x < y < 1$$
$$= 0, \quad \text{otherwise}$$

- Find the marginal density of X and Y .
- Are X and Y independent?
- Find $P(X < \frac{1}{2} \cap Y < \frac{1}{4})$ $3+1+4=8$

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UNIT—V

9. Answer any two of the following questions : $2 \times 2 = 4$

- State the properties of binomial distribution.
- State the conditions under which binomial distribution tends to Poisson distribution.
- Write the p.d.f. of exponential distribution with parameter θ .

10. Answer any one of the following questions : 10

- (i) Define binomial distribution. 2
- (ii) Find the m.g.f. of binomial distribution about origin. 3
- (iii) If $X \sim B(n, p)$, then show that

$$E\left(\frac{X}{n} - p\right)^2 = \frac{pq}{n}$$

Find the mean of Poisson distribution. $3+2=5$

- (i) Define Poisson distribution. Find the recurrence relation of Poisson distribution. $1+4=5$
- (ii) If $X \sim N(\mu, \sigma^2)$, then show that $z = \frac{X - \mu}{\sigma} \sim N(0, 1)$, symbols are of their usual meaning. 5

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