FYUG Even Semester Exam., 2025

STATISTICS

(4th Semester)

Course No. : STADSC–252

(Mathematical and Numerical Analysis)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) Show that for a singleton set, supremum is equal to infimum.

(b) Find the supremum and infimum of the set

$$S = \left\{ \frac{1}{n} ;\, n \in N \right\}$$

(c) Define neighbourhood of a point. Write any two properties of neighbourhood.

2. Answer any *one* of the following questions : 10

(a) (i) Define interior point and open set. Show that finite intersection of open sets is also an open set. 5

(ii) Show that union of finite collection of closed sets is closed. Also show that every finite subset of $\mathbb{R}$ is a closed set. 5

(b) (i) Define sequence, limit of sequence, convergent sequence and divergent sequence. 5

(ii) State and prove Cauchy's first theorem on limit. 5

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) State Cauchy's general principle of convergence of series.

(b) State Raabe's test.

(c) Define alternating series and infinite series.

4. Answer any *one* of the following questions : 10

(a) (i) State and prove Leibnitz's theorem on alternating series. 5

(ii) State D'Alembert's ratio test. Test for the convergence of the following series : 5

$$\frac{2}{1^2+1}+\frac{2^2}{2^2+1}+\frac{2^3}{3^2+1}+\cdots$$

(b) (i) State Cauchy's n-th root test. Test the convergence of the series : 5

$$\sum\left(\frac{n}{n+1}\right)^{n^2}$$

(ii) Test the convergence and absolute convergence of the following series : 5

$$\frac{1}{1\cdot 3}-\frac{1}{2\cdot 4}+\frac{1}{3\cdot 5}-\frac{1}{4\cdot 6}+\cdots$$

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) State Lagrange's mean value theorem.

(b) State Taylor's theorem with Lagrange's remainder.

(c) Write the expansions of $\cos x$ and $\log_e(1+x)$.

6. Answer any *one* of the following questions : 10

(a) (i) State and prove Rolle's theorem. 5

(ii) Obtain Maclaurin's series expansion of $\sin x$. 5

(b) (i) State and prove Cauchy's mean value theorem. 5

(ii) Verify Lagrange's mean value theorem for the function

$$f(x) = \sqrt{x^2 - 4} \text{ in } [2, 4]$$ 5

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Find the function u_x whose first difference is ${}^xC_n$.

(b) Evaluate

$$\left(\frac{\Delta^2}{E}\right)e^x$$

(c) State Newton-Gregory backward interpolation formula.

8. Answer any *one* of the following questions : 10

(a) (i) By means of Lagrange's interpolation formula, prove that

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 + y_1) - \frac{1}{2}(y_{-1} + y_{-3})\right]$$ 5

(ii) Show that nth order divided differences of a polynomial of degree n are constants. 5

(b) (i) State and prove Newton-Gregory forward interpolation formula. 5

(ii) Establish the following identity : 5

$$u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \cdots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n}$$

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Solve the difference equation

$$u_{x+1} - u_x = ca^x$$

where c is a periodic function of period 1.

(b) Write a note on numerical integration.

(c) State Gauss' backward central difference formula.

10. Answer any *one* of the following questions : 10

(a) *(i)* State and prove Gauss' forward central difference formula. 5

(ii) State and prove Stirling's interpolation formula. 5

(b) *(i)* Derive general quadrature formula of numerical integration and hence obtain Newton's 1/3rd rule. 6

(ii) Explain Newton-Raphson method for the solution of polynomial equations. 4

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