FYUG Even Semester Exam., 2025

STATISTICS

(4th Semester)

Course No. : STADSC–251

(Statistical Inference)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : $2 \times 2 = 4$

(a) Define estimate and estimator.

(b) Define consistency.

(c) What is factorization theorem of sufficiency?

2. Answer any *one* of the following questions : 10

(a) *(i)* Define unbiasedness.

Show that

$$\frac{\sum x_i (\sum x_i - 1)}{n(n-1)}$$

is an unbiased estimator of θ^2 for the sample $x_1, x_2, \cdots, x_n$, which takes the values 1 and 0 with respective probabilities θ and $(1-\theta)$. 1+3=4

(ii) If T_n is a consistent estimator of θ and $\psi(\theta)$ is a continuous function of θ, then show that $\psi(T_n)$ is a consistent estimator of $\psi(\theta)$. 3

(iii) If $x_1, x_2, \cdots, x_n$ be a random sample from a uniform population $[0, \theta]$, then find a sufficient estimator for θ. 3

(b) *(i)* Define efficient estimator. 1

(ii) What is sufficiency? 1

(iii) Let T_1 and T_2 be unbiased estimators of $\gamma(\theta)$ with efficiencies e_1 and e_2 respectively and $\rho = \rho_\theta$ be the correlation between them. Prove that

$$\sqrt{e_1 e_2} - \sqrt{(1-e_1)(1-e_2)} \leq \rho \leq \sqrt{e_1 e_2} + \sqrt{(1-e_1)(1-e_2)}$$ 5

(iv) A random sample x_1, x_2, x_3, x_4 and x_5 of size 5 is drawn from a normal population with unknown mean μ. Consider the following estimators to estimate μ :

$$T_1 = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}$$

$$T_2 = \frac{x_1 + x_2}{2} + x_3$$

$$T_3 = \frac{2x_1 + x_2 + \lambda x_3}{3}$$

where 'λ' is such that T_3 is an unbiased estimator of μ.

Find λ. Are T_1 and T_2 unbiased? State reasons which is the best amongst T_1, T_2 and T_3. 3

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Define MVUE.

(b) State Lehmann-Scheffe theorem.

(c) What is minimum variance bound estimator?

4. Answer any *one* of the following questions : 10

(a) *(i)* Write the regularity conditions of Crammer-Rao inequality. 2

(ii) Prove that the variance of an unbiased estimator T for $\gamma(\theta)$ satisfies the inequality

$$V(t) \geq \frac{[\gamma'(\theta)]^2}{E_\theta\left[\frac{\partial}{\partial\theta}\log L\right]^2}$$ 5

(iii) Obtain the MVB estimator for μ in normal population $N(\mu, \sigma^2)$, where σ^2 is known. 3

(b) *(i)* State and prove Rao-Blackwell theorem. 4

(ii) Define complete family of a distribution. 2

(iii) If $x_1, x_2, \cdots, x_n$ be a random sample from Bernoulli distribution

$$f(x, \theta) = \theta^x \cdot (1-\theta)^{1-x}, \; x = 0, 1$$

then show that $\sum_{i=1}^{n} x_i$ is a complete sufficient statistic for θ. 4

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Define critical region and level of significance.

(b) What is power of a test?

(c) Define likelihood function.

6. Answer any *one* of the following questions : 10

(a) *(i)* Define maximum likelihood estimator and state some assumptions of MLE. 2+2=4

(ii) Find the MLE for the parameter λ of a Poisson distribution on the basis of a sample of size n. 2

(iii) Find the MLE of the parameter α and λ (λ being large) of the distribution

$$f(x;\alpha,\lambda) = \frac{1}{\Gamma(\lambda)}\left(\frac{\lambda}{\alpha}\right)^{\lambda} \cdot e^{-\frac{\lambda x}{\alpha}} \cdot x^{\lambda-1},$$

$$\lambda > 0;\; 0 \leq x \leq \infty$$

given, for large values of λ

$$\psi(\lambda) = \frac{\partial}{\partial\lambda}\log\sqrt{\lambda} = \log\lambda - \frac{1}{2\lambda} \text{ and}$$

$$\psi'(\lambda) = \frac{1}{\lambda} + \frac{1}{2\lambda^2}$$ 4

(b) *(i)* Define methods of moment estimation. 2

(ii) Using the method of moments, estimate α and β for the following pdf :

$$f(x; \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \cdot x^{\alpha-1} \cdot e^{-\beta x}, \ 0 < x < \infty$$ 4

(iii) For the double Poisson distribution

$$P(X = x) = \frac{1}{2} \cdot \frac{e^{-m_1} \cdot m_1^x}{\lfloor x} + \frac{1}{2} \cdot \frac{e^{-m_2} \cdot m_2^x}{\lfloor x},$$

$$x = 0, 1, 2, \cdots$$

show that the estimators for m_1 and m_2 by the method of moments are

$$\mu_1' \pm \sqrt{\mu_2' - \mu_1' - \mu_1'^2}$$ 4

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Define most powerful test and uniformly most powerful test.

(b) Define most powerful critical region.

(c) State the properties of likelihood ratio test.

8. Answer any *one* of the following questions : 10

(a) (i) State and prove Neymann-Pearson lemma. 5

(ii) Find the Neymann-Pearson test of size α for testing $H_0 : \beta = 1$ against $\beta = \beta_1$ based on a sample of size 1 from

$$f(x) = \beta \cdot x^{\beta-1}, \ 0 < x < 1$$
$$= 0, \quad \text{otherwise}$$ 5

(b) (i) If $x \geq 1$ is the critical region for testing $H_0 : \theta = 2$, against the alternative hypothesis $H_1 : \theta = 1$, on the basis of the single observation from the population

$$f(x, \theta) = \theta \cdot e^{-\theta x}, \ \theta \leq x < \infty$$

then obtain the type-I and type-II errors. 5

(ii) Let x be a random variable having pdf

$$f(x, \theta) = \theta^{-1} \cdot e^{-\frac{x}{\theta}}, \ x \geq 0, \ \theta > 0$$

To test $H_0 : \theta = 2$ against $H_1 : \theta = 1$, use a random sample x_1 and x_2 of size 2 and define the critical region

$$w = \{(x_1, x_2) : 9 \cdot 5 \leq x_1 + x_2 < \infty\}$$

Find—

(1) power function and significant level of the test;

(2) the power when H_1 is true. 5

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) Define confidence interval and confidence limit.

(b) State the difference between point estimation and interval estimation.

(c) Obtain $100(1-\alpha)\%$ confidence interval of parameter θ for the normal distribution

$$f(x, \theta, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \quad -\infty < x < \infty$$

10. Answer any *one* of the following questions : 10

(a) (i) Show that for the distribution

$$dF = \theta \cdot e^{-\theta x}, \quad 0 < x < \infty$$

central confidence limits for θ for large sample with 95% confidence coefficient are given below

$$\theta = \frac{1 \pm \frac{1 \cdot 96}{\sqrt{n}}}{\bar{x}}$$

5

(ii) Given the observation from a population with pdf

$$f(x, \theta) = \frac{2}{\theta^2}(\theta - x), \quad 0 < x < \theta$$

obtain $100(1-\alpha)\%$ confidence interval for θ. 5

(b) Consider a random sample from pdf

(i) $f(x, \theta) = \theta \cdot x^{\theta-1}, \; 0 \le x \le 1, \; \theta > 0$
$\quad = 0, \quad$ otherwise

(ii) $f(x, \theta) = e^{-(x-\theta)}, \; x \ge 0, \; -\infty < \theta < \infty$
$\quad = 0, \quad$ otherwise

(iii) $f(x, \theta) = \frac{1}{\theta} \cdot e^{-\frac{x}{\theta}}, \; x > 0, \; \theta > 0$

Find in each case the $100(1-\alpha)\%$ level of confidence interval for θ. 10

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