FYUG Even Semester Exam., 2025

PHYSICS

(4th Semester)

Course No. : PHYDSC–251

(Mathematical Physics—II)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following : 2×2=4

(a) What happens to Fourier series expansion of a periodic function, if the function is even in nature?

(b) What are Dirichlet's conditions for a Fourier series expansion?

(c) How will you change the function $f(x)$ the interval $(-\pi, \pi)$ to $(-l, +l)$ in Fourier series?

2. Find the series of sines and cosines of multiples of x which represents $f(x)$ in the interval $-\pi < x < \pi$, where

$$f(x) = 0, \quad \text{when } -\pi < x \le 0$$
$$= \frac{\pi x}{4}, \quad \text{when } 0 < x \le \pi$$

and hence deduce $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots$.

5+5=10

OR

3. *(a)* Obtain a Fourier expression for

$$f(x) = x^3 \quad \text{for } -\pi < x < \pi$$ 5

(b) Obtain the complex form of the Fourier series of the function

$$f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ 1, & 0 \le x \le \pi \end{cases}$$ 5

UNIT—II

4. Answer any *two* from the following : 2×2=4

(a) Explain the terms 'ordinary point' and 'singular point' with reference to ordinary differential equation.

(b) Check whether $x = 0$ is an ordinary point or singular point of the following differential equations :

(i) $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 + 2)y = 0$

(ii) $(2 + x)\frac{d^2y}{dx^2} + (1 + x)\frac{dy}{dx} + 3y = 0$

(c) State Hermite and Laguerre differential equation of second order.

5. *(a)* Obtain a series solution in powers of x for differential equation

$$2x(1 - x)\frac{d^2y}{dx^2} + (1 - x)\frac{dy}{dx} + 3y = 0$$

by using Frobenius method. 5

(b) Applying Frobenius method to solve Hermite differential equation

$$\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2ny = 0$$

where n is an integer. 5

OR

6. *(a)* State and explain the generating function of Hermite polynomials. 5

(b) Show that Rodrigue's formula for Laguerre polynomials is

$$L_n(x) = \frac{e^x}{\lfloor n} \frac{d^n}{dx^n}(x^n e^{-x})$$

where n is an integer. 5

UNIT—III

7. Answer any *two* from the following : 2×2=4

(a) Write the Rodrigue's formula for Legendre polynomial. What is the orthogonality condition of the Legendre polynomial?

(b) Using the generating function of $J_n(x)$ to find the values of $J_0(x)$ and $J_1(x)$.

(c) Show that $P_n(-x) = (-1)^n P_n(x)$.

8. *(a)* Show that $\int_{-1}^{+1} [P_n(x)]^2 \, dx = \frac{2}{2n+1}$. 5

(b) Prove the following recurrence relations for Legendre polynomial : 3+2=5

(i) $nP_n(x) = xP_n'(x) - P_{n-1}'(x)$

(ii) $(1 - x^2) P_n'(x) = n[P_{n-1}(x) - xP_n(x)]$

OR

9. *(a)* If α and β are different roots of $J_n(x) = 0$, then show that

$$\int_0^1 x J_n(\alpha x) J_n(\beta x) \, dx = 0 \quad \text{for} \quad \alpha \neq \beta$$ 5

(b) Show that

$$J_{3/2}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right)$$ 5

UNIT—IV

10. Answer any *two* from the following : 2×2=4

(a) Write down the conditions for Laplace transform to exist.

(b) Explain two properties of Laplace transform.

(c) What do you mean by derivative of Laplace transform?

11. *(a)* Find the Laplace transform of the following functions : 3+2=5

(i) $f(t) = t^n$

(ii) $f(t) = \sin^2 t$

(b) Find the Laplace transform of $e^{-at} \sin bt$. 5

OR

12. (a) State and explain change of scale theorem. 3

(b) Given $L\left(2\sqrt{\frac{t}{\pi}}\right) = \frac{1}{s^{3/2}}$, show that

$$L\left(\frac{1}{\sqrt{\pi t}}\right) = \frac{1}{\sqrt{s}}$$ 2

(c) Find the Laplace transform of the wave-form $f(t) = \frac{2t}{3}$ $(0 \le t \le 3)$. 5

UNIT—V

13. Answer any *two* from the following : 2×2=4

(a) What do you mean by inverse Laplace transform?

(b) State convolution theorem for inverse Laplace transform.

(c) Find the inverse Laplace transform of $\frac{6}{s^2+36}$.

14. (a) Use convolution theorem to find

$$L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$$ 5

(b) Find the solution of damped harmonic oscillator using Laplace transforms. 5

OR

15. (a) Find the inverse Laplace transform of

$$\frac{1}{(s+1)(s^2+1)}$$ 5

(b) Using Laplace transforms, find the solution of coupled differential equations of first-order. 5

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