

**2021/TDC/CBCS/ODD/
PHSHCC-101T/147**

**TDC (CBCS) Odd Semester Exam., 2021
held in March, 2022**

PHYSICS

(1st Semester)

Course No. : PSHCC-101T

(Mathematical Physics—I)

Full Marks : 50

Pass Marks : 20

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

SECTION—A

Answer any *ten* of the following questions : $2 \times 10 = 20$

1. Find the values of x , y and z which satisfy the matrix equation

$$\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$$

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2. If

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -2 \\ -1 & 0 \\ 2 & -1 \end{bmatrix}$$

then obtain the product AB .

3. What do you mean by 'order' and 'degree' of a differential equation?

4. Find the area of the parallelogram whose adjacent sides are $\hat{i} - 2\hat{j} + 3\hat{k}$ and $2\hat{i} + \hat{j} - 4\hat{k}$.

5. Find the volume of the parallelopiped if $\vec{a} = -3\hat{i} + 7\hat{j} + 5\hat{k}$, $\vec{b} = -3\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{c} = 7\hat{i} - 5\hat{j} - 3\hat{k}$ are the three coterminous edges of the parallelopiped.

6. At any point of the curve $x = 3 \cos t$, $y = 3 \sin t$, $z = 4t$, find the tangent vector.

7. If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in the xy -plane from $(0, 0)$ to $(1, 4)$ along a curve $y = 4x^2$, then find the work done.

8. Evaluate by Stokes' theorem

$$\oint_C (yzdx + zxdy + xydz)$$

where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

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9. State Green's theorem.

10. Write the expression for line element in spherical polar coordinate system.

11. Write the expression for volume element in cylindrical coordinate system.

12. Write the expression for gradient of a scalar function in cylindrical coordinate system.

13. Define the term 'constant error' and give a suitable example.

14. The temperatures of two bodies measured by a thermometer are given by $T_1 = (20 \pm 0.5)^\circ\text{C}$ and $T_2 = (50 \pm 0.5)^\circ\text{C}$. Calculate the temperature difference and error therein.

15. What do you mean by least square fit method?

SECTION—B

Answer any five of the following questions : $6 \times 5 = 30$

16. (a) If

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

then show that $A^2 - 4A - 5I = 0$ where I and O are unit matrix and null matrix of order 3 respectively.

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(4)

(b) If

$$A = \frac{1}{9} \begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$$

then prove that $A^{-1} = A'$, A' being the transpose of A .

17. (a) Solve the following differential equation by the method of integrating factor

$$(x^3 - x) \frac{dy}{dx} - (3x^2 - 1)y = x^5 - 2x^3 + x$$

- (b) Solve the differential equation

$$(2xy + x^2)dy = (3y^2 + 2xy)dx$$

18. (a) Find m so that the vectors $2\hat{i} - 4\hat{j} + 5\hat{k}$, $\hat{i} - m\hat{j} + \hat{k}$ and $3\hat{i} + 2\hat{j} - 5\hat{k}$ are coplanar.

- (b) Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Find the vector $\vec{a} \times (\vec{b} \times \vec{c})$.

19. (a) If $\frac{d\vec{a}}{dt} = \vec{u} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{u} \times \vec{b}$, then prove that $\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{u} \times (\vec{a} \times \vec{b})$.

- (b) If $\phi = 3x^2y - y^3z^2$, then find grad ϕ at the point $(1, -2, -1)$.

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20. (a) If $\vec{F} = 2y\hat{i} - z\hat{j} + x\hat{k}$, then evaluate $\int_C \vec{F} \times d\vec{r}$ along the curve $x = \cos t$, $y = \sin t$, $z = 2\cos t$ from $t = 0$ to $t = \frac{\pi}{2}$.

- (b) A vector field is given by $\vec{F} = (\sin y)\hat{i} + x(1 + \cos y)\hat{j}$. Evaluate the line integral over a circular path $x^2 + y^2 = a^2$, $z = 0$.

21. (a) Using Green's theorem, evaluate $\int_C (x^2y dx + x^2 dy)$ where C is the boundary described counter-clockwise of the triangle with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$.

- (b) Using Stokes' theorem, evaluate $\int_C [(2x - y)dx - yz^2 dy - y^2 z dz]$ where C is the circle $x^2 + y^2 = 1$ corresponding to the surface of the sphere of unit radius.

22. (a) Find the expression for $\nabla^2 \phi$ in orthogonal curvilinear coordinate system.

- (b) If $u = 2x + 3$, $v = y - 4$, $w = z + 2$ and $\vec{r}$ be the position vector, i.e., $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that $\frac{\partial \vec{r}}{\partial u}$, $\frac{\partial \vec{r}}{\partial v}$ and $\frac{\partial \vec{r}}{\partial w}$ are mutually orthogonal.

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23. (a) Use the Newton-Raphson method to find a root of the equation $x^3 - 2x - 5 = 0$.

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- (b) Evaluate

$$I = \int_0^1 \frac{1}{1+x} dx$$

correct to three decimal places using Simpson rule with $h = 0.125$.

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24. (a) Discuss the different types of systematic errors associated with a measurement.

3

- (b) What is random error?

The period of oscillation of a simple pendulum is

$$T = 2\pi \sqrt{\frac{L}{g}}$$

The measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 sec using a wrist watch of 1 sec resolution. What is the accuracy in the determination of g ? $1+2=3$

25. (a) Define standard error and probable error. What are the rules of testing the significance of correlation?

$2+1=3$

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(7)

- (b) If two resistors of resistances

$$R_1 = (5 \pm 0.1) \text{ ohm and } R_2 = (10 \pm 0.2) \text{ ohm}$$

are connected in (i) series and (ii) parallel, then find the equivalent resistance in each case with limits of percentage error.

$$1\frac{1}{2} + 1\frac{1}{2} = 3$$

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