2025/FYUG/ODD/SEM/
MTMDSM301/302T/075

**FYUG Odd Semester Exam., 2025**

# MATHEMATICS

**( 5th Semester )**

Course No. : MTMDSM301/302T

**( Geometry and Vectors )**

*Full Marks : 70*
*Pass Marks : 28*

*Time : 3 hours*

*The figures in the margin indicate full marks for the questions*

## UNIT—I

1. Answer any *two* of the following questions : $2\times2=4$

(a) Find the transformed equation of the curve $x^2+2\sqrt{3}\,xy-y^2=8$, when the axes are rotated through an angle of $30^\circ$.

(b) Find the value of $k$ for which the equation

$$2x^2+3xy-2y^2+7x+y+k=0$$

may represent a pair of straight lines.

( Turn Over )



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(c) If the pair of straight lines $x^2 - 2pxy - y^2 = 0$ and $x^2 - 2qxy - y^2 = 0$ be such that each pair bisects the angles between the other pair, then prove that $pq + 1 = 0$.

2. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) If by rotation of rectangular axes without changing the origin, the expression $ax^2 + 2hxy + by^2$ changes to $a'x'^2 + 2h'x'y' + b'y'^2$, then prove that $a + b = a' + b'$ and $ab - h^2 = a'b' - h'^2$ 6

(b) Show that the equation

$$16x^2 + 24xy + 9y^2 + 40x + 30y - 75 = 0$$

represents a pair of parallel straight lines and find the distance between them. 4

(c) Prove that the product of the perpendiculars from the point $(\alpha, \beta)$ on the line $ax^2 + 2hxy + by^2 = 0$ is

$$\frac{a\alpha^2 + 2h\alpha\beta + b\beta^2}{\sqrt{(a-b)^2 + 4h^2}}$$

5

(d) Find the equation of the bisectors of the angles between the lines given by

$$ax^2 + 2hxy + by^2 = 0$$



(Continued)



UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Find the condition that the line $lx + my = n$ is a tangent to the circle $x^2 + y^2 = a^2$.

(b) Prove that the line

$$y = m(x + a) + \frac{a}{m}$$

touches the parabola $y^2 = 4a(x + a)$.

(c) Show that the line $x + 2y = 4$ touches the ellipse $3x^2 + 4y^2 = 12$.

4. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Find the equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at the point $(\alpha, \beta)$ on it. 5

(b) Prove that the straight line $lx + my + n = 0$ is a normal to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

if

$$\frac{a^2}{l^2} - \frac{b^2}{m^2} = \frac{(a^2 + b^2)^2}{n^2}$$

5

( Turn Over )

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(c) Show that three normals can be drawn from an external point to the parabola $y^2 = 4ax$. 5

(d) Show that the product of the lengths of the perpendiculars drawn from the foci on any tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $b^2$. 5

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Find the pole of the line $lx + my + n = 0$ with respect to the parabola $y^2 = 4ax$.

(b) Find the nature of the conic $\frac{8}{r} = 4 - 5\cos\theta$.

(c) Find the point on the curve $\frac{14}{r} = 3 - 8\cos\theta$ whose radius vector is 2.

6. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] :

(a) If the polar of a point $P$ with respect to a conic passes through a point $Q$, then prove that the polar of the point $Q$ with respect to the same conic passes through $P$. 5



( Continued )



(b) If $PSP'$ and $QSQ'$ be any two perpendicular focal chords of a conic, then show that

$$\frac{1}{SP \cdot SP'} + \frac{1}{SQ \cdot SQ'}$$

is a constant. 5

(c) Find the pole of the line $4x + 15y - 2 = 0$ with respect to the conic

$$x^2 + 3xy + 4y^2 - 5x + 3 = 0$$ 5

(d) Find the polar equation of a conic of the form $\frac{l}{r} = 1 + e\cos\theta$. Write down the equation of directrix. 5

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Find the value of $p$ so that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + p\hat{j} + 5\hat{k}$ are coplanar.

(b) Prove that

$$\vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = \vec{0}$$

( Turn Over )



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*(c)* Write the vector equation of a sphere—

*(i)* whose centre is $\vec{c}$ and radius $\alpha$;

*(ii)* whose ends of one of its diameter are $\vec{a}$ and $\vec{b}$.

**8.** Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

*(a)* Prove that—

*(i)* $[\vec{a}+\vec{b} \;\; \vec{b}+\vec{c} \;\; \vec{c}+\vec{a}] = 2[\vec{a} \;\; \vec{b} \;\; \vec{c}]$

*(ii)* $\hat{i}\times(\vec{a}\times\hat{i})+\hat{j}\times(\vec{a}\times\hat{j})+\hat{k}\times(\vec{a}\times\hat{k}) = 2\vec{a}$ 3+3=6

*(b)* Prove that the necessary and sufficient condition for a vector $\vec{r}=\vec{f}(t)$ to have a constant magnitude is $\vec{f}\cdot\frac{d\vec{f}}{dt}=0$. 4

*(c)* *(i)* Prove that

$$\vec{a}\times(\vec{b}\times\vec{a}) = (\vec{a}\times\vec{b})\times\vec{a}$$ 2

*(ii)* Find the vectorial equation of a line which passes through a given point and is parallel to a given vector. 4

*(d)* If

$$\vec{A} = 5t^2\hat{i}+t\hat{j}-t^3\hat{k}$$
$$\vec{B} = \sin t\hat{i} - \cos t\hat{j}$$

then obtain $\frac{d}{dt}(\vec{A}\cdot\vec{B})$ and $\frac{d}{dt}(\vec{A}\times\vec{B})$. 4





## UNIT—V

Answer any *two* of the following questions : 2×2=4

*(a)* Prove that $\text{grad}(\vec{r}\cdot\vec{a}) = \vec{a}$, where $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$, $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$.

*(b)* If

$$\vec{f} = xy^2\hat{i}+2x^2yz\hat{j}-3yz^2\hat{k}$$

then find div $\vec{f}$ at (1, −1, 1).

*(c)* Find curl $\vec{v}$, where $\vec{v} = 2xy\hat{i}-yz\hat{j}+z\hat{k}$.

Answer *either* [*(a)* and *(b)*] *or* [*(c)* and *(d)*] :

*(a)* If $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$, then prove that 5

$$\vec{\nabla}r^n = nr^{n-2}\vec{r}$$

*(b)* Show that curl grad $\phi = 0$, where 5

$$\phi = x^2y+2xy+z^2$$

*(c)* Find div grad $\phi$, where $\phi = 2x^2y^3z^4$. 5

*(d)* If $\vec{v} = \vec{w}\times\vec{r}$, where $\vec{w}$ is a constant vector, then prove that curl $\vec{v} = 2\vec{w}$. 5

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