2025/FYUG/ODD/SEM/
MTMDSC-303T/074

FYUG Odd Semester Exam., 2025

MATHEMATICS

(5th Semester)

Course No. : MTMDSC-303T

(Real Analysis—II)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* from the following : 2×2=4

(a) Define finite set.

(b) Justify that the product of two uniformly continuous functions is uniformly continuous.

(c) Define Lipschitz function.

2. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Show that the set of integers is countable. 5

(Turn Over)

(b) Show that a continuous function on a closed and bounded interval is uniformly continuous.

(c) Show that the open interval (0, 1) is uncountable.

(d) Show that $f(x) = \frac{1}{x}$ is uniformly continuous on $[a, \infty)$, where $a > 0$ but fails to be so on $(0, \infty)$.

UNIT—II

3. Answer any *two* from the following : 2×2=

(a) Define convergence of a series.

(b) Check if the series is convergent :

$$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\ldots$$

(c) What is a conditionally convergent series?

4. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Show that the harmonic series $\Sigma\frac{1}{n}$ is divergent. 5

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Test the convergence of the following : 2+3=5

(i) $\sum \frac{1}{n^2 - n + 1}$

(ii) $\sum \frac{n}{2^n}$

Show that every absolutely convergent series is convergent. Give example, with justification, to show that the converse is not true, in general. 4+1=5

(d) Test the convergence of the following : 2+3=5

(i) $\sum \frac{(-1)^{n+1}}{n^2+1}$

(ii) $\sum (-1)^n \frac{(n+1)^n}{n^n}$

UNIT—III

Answer any *two* from the following : 2×2=4

(a) Let $f : [a, b] \to \mathbb{R}$ be bounded and P be any partition of $[a, b]$. Show that

$$L(P, f) \le U(P, f)$$

(b) Show that a constant function is Darboux integrable.

(Turn Over)

(c) Give example, with justification, of a function that is not Darboux integrable on [0, 1].

6. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Let $f:[a, b]\to\mathbb{R}$ be bounded. Let P and Q be partitions of $[a, b]$ and Q is a refinement of P. Show that

$$L(P, f)\le L(Q, f) \text{ and } U(Q, f)\le U(P, f)$$

(b) Let $f:[a, b]\to\mathbb{R}$ and $g:[a, b]\to\mathbb{R}$ be Darboux integrable. Show that $f+g$ is Darboux integrable and

$$\int_a^b (f+g)\,dx=\int_a^b f\,dx+\int_a^b g\,dx$$

(c) Let $f:[a, b]\to\mathbb{R}$ be bounded. Show that f is integrable if and only if for every $\varepsilon>0$ there exists a partition P of $[a, b]$ such that $U(P, f)-L(P, f)<\varepsilon$.

(d) If f is bounded and integrable on $[a, b]$, show that $|f|$ is also bounded and integrable on $[a, b]$. Also show that

$$\left|\int_a^b f\,dx\right|\le\int_a^b |f|\,dx$$

UNIT—IV

7. Answer any *two* from the following : 2×2=4

(a) Define tagged partition of a closed interval. Give an example.

(b) Define a Riemann integrable function.

(c) Calculate the Riemann sum for $f(x)=x^2$ in [0, 4] for the partition $P=\{0, 1, 2, 4\}$ with tags at the left end points of the subintervals.

8. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Show that if f is Riemann integrable on $[a, b]$, then the value of the integral is unique. 4

(b) Show that every Riemann integrable function is bounded. Give example of a bounded function that is not integrable. 5+1=6

(c) Show that every continuous function on $[a, b]$ is Riemann integrable on $[a, b]$. 5

(d) If $f(x)\le g(x)\ \forall\ x\in[a, b]$, then show that 5

$$\int_a^b f\le\int_a^b g$$

(Turn Over)

UNIT—V

9. Answer any *two* from the following : 2×2=4

(a) If f is continuous on $[a, b]$, then show that there exists a number $c \in [a, b]$ such that

$$\int_a^b f\,dx = f(c)(b-a)$$

(b) Examine the convergence of

$$\int_0^1 \frac{dx}{x^2}$$

(c) Examine the convergence of

$$\int_0^\infty \frac{x\,dx}{1+x^2}$$

10. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) If a function f is bounded and integrable on $[a, b]$ and

$$F(x) = \int_a^x f(t)\,dt, \quad a \le x \le b$$

then, show that F is continuous on $[a, b]$. 4

(b) Test the convergence of the following : 3+3=6

(i) $\int_a^b \frac{dx}{(x-a)^n}$

(ii) $\int_0^{\pi/2} \frac{\sin x}{x^p}$

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(c) If f is bounded and integrable on $[a, b]$ and there exists a function F such that $F'(x) = f(x)$ for all $x \in [a, b]$, then show that

$$\int_a^b f\,dx = F(b) - F(a)$$ 5

(d) Show that

$$\int_0^\infty x^{m-1} e^{-x}\,dx$$

converges if and only if $m > 0$. 5

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