

**2025/FYUG/ODD/SEM/
MTMDSC-301T/072**

FYUG Odd Semester Exam., 2025

MATHEMATICS

(5th Semester)

Course No. : MTMDSC-301T

(Partial Differential Equations)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

UNIT—I

Answer any *two* of the following questions :

$2 \times 2 = 4$

- (a) Eliminate the arbitrary constants a and b from $z = (x - a)^2 + (y - b)^2$ to form a partial differential equation.
- (b) Form a partial differential equation by eliminating the arbitrary function f from the relation $z = x + y + f(xy)$.
- (c) Solve the equation $\frac{\partial^2 z}{\partial x \partial y} = 0$.

(2)

2. Answer either [(a) and (b)] or [(c) and (d)] :

(a) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x+y+z, x^2+y^2-z^2)=0$.

(b) Find the integral surface of the linear partial differential equation

$$x(y^2+z)\frac{\partial z}{\partial x} - y(x^2+z)\frac{\partial z}{\partial y} = (x^2-y^2)z$$

which contains the straight line $x+y=0, z=1$.

(c) Solve the PDE

$$\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z}{\partial y} = 1$$

subject to the conditions $u(x, 0)=0,$

$$\frac{\partial u}{\partial y}(0, y) = \sin y.$$

(d) Solve :

$$(x^2+2y^2)p - xyq = xz$$

UNIT—II

3. Answer any two of the following questions :

2×2=4

(a) Write down the Charpit's auxiliary equations of $q=3p^2$.

(3)

(b) Find a complete integral of the PDE $z=pq$.

(c) Solve :

$$z = px + qy + pq$$

Answer either [(a) and (b)] or [(c) and (d)] : 10

(a) Show that the partial differential equations $xp-yq=x$ and $x^2p+q=xz$ are compatible and find their solution. 5

(b) Find a complete integral of the partial differential equation 5

$$(x^2-y^2)pq - xy(p^2-q^2) = 1$$

(c) Find a complete and singular solution of the partial differential equation $(p^2+q^2)y=qz$. 5

(d) Solve :

$$z = px + qy + c\sqrt{1+p^2+q^2}$$

UNIT—III

5. Answer any two of the following questions : 2×2=4

(a) Solve :

$$x \frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial x}$$

(Turn Over)

(4)

(b) Solve :

$$(D^2 + 2DD' + D'^2)z = 0$$

(c) Find the particular integral of the PDE
 $(D^2 + 3DD' + 2D'^2)z = x + y$

6. Answer either [(a) and (b)] or [(c) and (d)] :

(a) Solve :

$$(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x + y)$$

(b) Solve :

$$(i) (D^2 - D'^2 + D - D')z = 0$$

$$(ii) t + s + q = 0$$

(c) Solve :

$$(D^2 - 2DD' - 15D'^2)z = 12xy$$

(d) Solve the Cauchy problem

$$\frac{\partial^2 z}{\partial y^2} - 2z = 0$$

$$u(x, 0) = x^3, \frac{\partial u}{\partial y}(x, 0) = e^{-x}$$

(5)

UNIT—IV

Answer any two of the following questions : $2 \times 2 = 4$

(a) Describe the classification of the linear second order PDE

$$A \frac{\partial^2 z}{\partial x^2} + B \frac{\partial^2 z}{\partial x \partial y} + C \frac{\partial^2 z}{\partial y^2} + D \frac{\partial z}{\partial x} + E \frac{\partial z}{\partial y} + Fz = G$$

(b) Find the canonical form of the partial differential equation

$$2 \frac{\partial z}{\partial x} + 3 \frac{\partial z}{\partial y} + 8z = 0$$

(c) Solve the PDE $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ using the method of separation of variables.

7. Answer either [(a) and (b)] or [(c) and (d)] : 10

(a) Obtain the canonical form of the PDE
 $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} + xyz = 1$ 5

and hence solve it.

(b) Solve the PDE

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial y} + 2z$$

using the method of separation of variables. 5

(Turn Over)

(6)

- (c) Reduce the PDE

$$\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$$

into canonical form and hence solve it.

- (d) Solve the PDE

$$\frac{\partial^2 z}{\partial x^2} - 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$$

using the method of separation of variables.

UNIT—V

9. Answer any two of the following questions : 2×2=4

- (a) Define homogeneous and non-homogeneous boundary conditions of a second-order PDE.

- (b) Give physical interpretation of the variables x , t and u in the wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

- (c) Find the eigenvalues of the differential equation

$$\frac{d^2 y}{dx^2} - \lambda^2 y = 0$$

(7)

Answer either [(a) and (b)] or [(c) and (d)] : 10

- (a) Derive heat conduction equation for a bar. 5

- (b) Describe Fourier half-range sine and cosine series. 5

- (c) Derive the one-dimensional homogeneous wave equation. 5

- (d) Solve completely the heat conduction equation

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \quad t > 0$$

subject to the initial condition

$$u(x, 0) = f(x), \quad 0 < x < 1$$

and boundary conditions

$$u(0, t) = 0; \quad u(1, t) = 0, \quad t > 0$$

5
