2025/FYUG/ODD/SEM/
STADSM–201T/040

FYUG Odd Semester Exam., 2025

STATISTICS

(3rd Semester)

Course No. : STADSM–201T

ndom Variables and Probability Distribution)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

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UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) Define random variable. What is discrete random variable?

(b) Define distribution function and state one of its properties.

(c) Define probability mass function (p.m.f.) and probability density function (p.d.f.).

(*Turn Over*)

2. Answer any *one* of the following questions : 10

(a) (i) A random variable X has the following distribution :

X	0	1	2	3	4	5	6	7	8
$P(X)$	K	$3K$	$5K$	$7K$	$9K$	$11K$	$13K$	$15K$	$17K$

1. Determine the value of K.
2. Find $P(X < 3)$.
3. Find $P(0 < X < 5)$.

2+2+1=5

(ii) Let X be a random variable such that

$$P(X = -2) = P(X = -1),\ P(X = 2) = P(X = 1)$$
and $P(X > 0) = P(X < 0)$.

Obtain the probability mass function of X and its distribution.

(b) (i) A continuous random variable has a p.d.f. $f(x) = 3x^2,\ 0 \le x \le 1$. Find a and b such that

1. $P(X \le a) = P(X > a)$
2. $P(X > b) = 0 \cdot 05$

3+2=5

(ii) A random process gives measurement of X between 0 and 1 with a p.d.f.

$$f(x) = \begin{cases} 12x^3 - 21x^2 + 10x, & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

1. Find $P\left(X \le \frac{1}{2}\right)$.

2. Find $P\left(X > \frac{1}{2}\right)$. 3+2=5

UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Define expectation and state some of its properties.

(b) Define moment generating function and state one of its properties.

(c) Show that $E(C) = C$.

4. Answer any *one* of the following questions : 10

(a) (i) Show that $E(X + Y) = E(X) + E(Y)$, where X and Y are random variables. 5

(ii) Show that

$$M_{X_1 + X_2}(t) = M_{X_1}(t) \cdot M_{X_2}(t)$$

where X_1 and X_2 are independent. 5

(b) (i) Two unbiased dice are thrown. Find the expected values of the sum of number of points on them. 5

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(ii) Let X be a random variable with the following probability distribution :

X	:	-3	6	9
$P(X)$	:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find $E(X)$, $E(X^2)$ and $V(3X+5)$.

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(a) Define joint probability mass function.

(b) Define marginal and conditional distribution function.

(c) Define independence of random variable.

6. Answer any *one* of the following questions : 10

(a) Let X and Y be two random variables each taking three values -1, 0, 1 and having the joint probability distribution

$X \rightarrow$ / $Y \downarrow$	-1	0	1
-1	0	0·1	0·1
0	0·2	0·2	0·2
1	0	0·1	0·1

(i) Find $E(X)$ and $E(Y)$.

(ii) Find $V(X)$, $V(Y)$. 5+5=10

(Continued)

(b) Two random variables X and Y have the following probability density functions :

$$f(x, y) = \begin{cases} 2 - x - y, & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find—

(i) marginal p.d.f. of X and Y;

(ii) conditional p.d.f. of X and Y;

(iii) variance of X. 4+4+2=10

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Find the mean of binomial distribution with parameters n and p.

(b) Find the mean of Poisson distribution with parameter λ.

(c) State the properties of normal distribution.

8. Answer any *one* of the following questions : 10

(a) (i) If $X_1 \sim B(n_1, p)$ and $X_2 \sim B(n_2, p)$, then show that 5

$$X_1 + X_2 \sim B(n_1 + n_2, p).$$

(Turn Over)

(ii) Show that Poisson distribution is a limiting case of binomial distribution. 5

(b) (i) State and prove the additive property of Poisson distribution. 5

(ii) If X and Y are independent Poisson variates, then show that conditional distribution of X given $X+Y$ is binomial. 5

UNIT—V

9. Answer any *two* of the following questions : $2\times2=4$

(a) What is convergence in distribution?

(b) State Chebyshev's inequality.

(c) Define convergence in probability.

10. Answer any *one* of the following questions : 10

(a) (i) State and prove weak law of large numbers. 6

(ii) A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes. 4

(b) (i) State and prove Bernoulli's law of large numbers. 6

(Continued)

(ii) How large a sample must be taken in order that probability will be at least 0·95 that $\bar{x}_n$ will be within μ? (μ is unknown and $\sigma = 1$). 4

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