

**2025 / FYUG / ODD / SEM /
STADSC-202T / 039**

FYUG Odd Semester Exam., 2025

STATISTICS

(3rd Semester)

Course No. : STADSC-202T

Unit Theorems and Sampling Distributions)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

UNIT—I

Answer any two of the following questions :

2×2=4

- (a) Define convergence in probability.
- (b) State Chebyshev's inequality.
- (c) Define almost sure convergence.

Answer any one of the following questions : 10

- (a) Write central limit theorem. State and prove De Moivre-Laplace central limit theorem. **3+7=10**

(Turn Over)

(2)

- (b) (i) State and prove Bernoulli's law of large numbers.
 (ii) State Lyapunov Central Limit Theorem.
 (iii) If $x_1, x_2, \dots$ be a Poisson variate with parameter λ . Use CLT to estimate $P(120 \leq S_n \leq 160)$, where $S_n = X_1 + X_2 + \dots + X_n$, $\lambda = 2$ and $n = 75$.

UNIT—II

3. Answer any two of the following questions :

- (a) Define parameter and statistic.
 (b) Explain sampling distribution of a statistic.
 (c) Write the probability density function of an r th order statistic.

4. Answer any one of the following questions :

- (a) Derive the p.d.f. and c.d.f. of the smallest and the largest order statistic.
 (b) Define standard error of a statistic. Explain its utility in tests of significance for large samples. Obtain standard error of sample mean, when sample is drawn in SRSWOR.

(Continued)

UNIT—III

Answer any two of the following questions :

- (a) Define null and alternative hypotheses.
 (b) Define type-I and type-II errors.
 (c) Define level of significance.

Answer any one of the following questions :

- (a) Write a short note on large sample test in test of significance. Describe the large sample test for single mean.
 (b) Describe the test of significance for large samples for difference of two means and difference of standard deviations for two distinct populations.

UNIT—IV

Answer any two of the following questions :

- (a) Define χ^2 -variate and state the characteristic function of χ^2 -distribution.
 (b) Write the p.d.f. of χ^2 -distribution. State its mean and variance.

(Turn Over)

(4)

- (c) State the conditions of validity of χ^2 -test.

8. Answer any one of the following questions :

- (a) (i) Derive the cumulant generating function (CGF) of χ^2 -distribution. Hence find the mean and variance of χ^2 -distribution.

- (ii) If χ_1^2 and χ_2^2 are independent variates with n_1 and n_2 degrees of freedom and

$$u = \frac{\chi_1^2}{\chi_1^2 + \chi_2^2}, \quad v = \chi_1^2 + \chi_2^2$$

are independently distributed, then prove that

$$\frac{\chi_1^2}{\chi_1^2 + \chi_2^2} \sim \beta_1\left(\frac{n_1}{2}, \frac{n_2}{2}\right) \text{ and}$$

$$\chi_1^2 + \chi_2^2 \sim \chi^2(n_1 + n_2)$$

- (b) (i) If X_i follows $\chi_{(n_i)}^2$, $i = 1, 2, \dots, k$, then

prove that $\sum_{i=1}^n X_i$ follows χ^2 -

distribution with $\sum_{i=1}^k n_i$ degrees of freedom.

(Continued)

(5)

- (ii) If X is a χ^2 -variate with n degrees of freedom, then prove that for large n $\sqrt{2X} \sim N(\sqrt{2n}, 1)$.

5

UNIT—V

9. Answer any two of the following questions :

2×2=4

- (a) Prove that for large n , student's t -distribution tends to standard normal distribution.
- (b) Discuss the relationship between student's t -distribution and F -distribution.
- (c) What are the assumptions of t -test for difference between means?

10. Answer any one of the following questions : 10

- (a) Derive the p.d.f. of student's t -distribution and hence derive its mean and variance.
- (b) Derive the p.d.f. of Snedecor's F -distribution with (n_1, n_2) degrees of freedom and hence find its mean and variance.

2025/FYUG/ODD/SEM/
STADSC-202T/039

26J—220/632