2025/FYUG/ODD/SEM/
STADSC–201T/038

FYUG Odd Semester Exam., 2025

STATISTICS

(3rd Semester)

Course No. : STADSC–201T

(Algebra)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) Define idempotent matrix and trace of a matrix.

(b) Define Hermitian and skew-Hermitian matrices.

(c) Define unitary and involutory matrices.

(*Turn Over*)

2. Answer any one of the following questions :

(a) (i) Prove that a necessary and sufficient condition for a matrix A to be symmetric is that A and A' are equal.

(ii) If $AB = A$ and $BA = B$, then $B^{-1}A^{-1} = A^{-1}$ and $A^{-1}B^{-1} = B^{-1}$ and hence prove that A^{-1} and B^{-1} are idempotent matrices.

(iii) Prove that any Hermitian matrix A can be written as $A = B + iC$, where B is a real symmetric and A and C are real skew symmetric.

(b) (i) If A^{θ} and B^{θ} be the transpose conjugates of the matrices A and B respectively, then prove that

$$(A^{\theta})^{\theta} = A, \; (A+B)^{\theta} = A^{\theta} + B^{\theta},$$
$$(AB)^{\theta} = B^{\theta}A^{\theta} \text{ and } (kA)^{\theta} = \bar{k}A^{\theta}$$

k being any complex number.

1+1+2+2=

(ii) Prove that a necessary and sufficient condition for a square matrix A to possess an inverse is that $|A| \neq 0$.

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UNIT—II

3. Answer any *two* of the following questions : 2×2=4

(a) Define rank of a matrix.

(b) Define echelon form of a matrix.

(c) Find the rank of a unit matrix of order 3.

4. Answer any *one* of the following questions : 10

(a) (i) Prove that rank of the transpose of a matrix is same as the original matrix. 5

(ii) Prove that

$$\begin{vmatrix} x & a & a & a \\ a & x & a & a \\ a & a & x & a \\ a & a & a & x \end{vmatrix} = (x+3a)(x-a)^3$$ 5

(b) (i) If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that 4

$$A^2 - 4A - 5I = 0.$$

(Turn Over)

(ii) Reduce the matrix

$$A = \begin{bmatrix} 1 & -1 & 2 & -3 \\ 4 & 1 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 0 & 1 & 0 & 2 \end{bmatrix}$$

to the normal form $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ and hence determine its rank.

UNIT—III

5. Answer any *two* of the following questions : 2×2

(a) Define characteristic roots and characteristic vectors.

(b) Prove that if X is a characteristic vector of a matrix A, then X cannot correspond to more than one characteristic value of A.

(c) Define quadratic form of a matrix.

6. Answer any *one* of the following questions : 10

(a) (i) Determine the characteristic roots and the corresponding characteristic vectors of the matrix 6

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(ii) Solve completely the following system of equations : 4

$$x+3y-2z=0$$
$$2x-y+4z=0$$
$$x-11y+14z=0$$

(b) (i) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

and verify that it is satisfied by A and hence obtain A^{-1}. 6

(ii) Check the consistency of the following equations : 4

$$x+y+z=-3$$
$$3x+y-2z=-2$$
$$2x+4y+7z=7$$

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Define ring with an example.

(b) Define linear dependence and linear independence of vector.

(c) Define vector space.

8. Answer any *one* of the following questions : 10

(a) (i) Let $V(F)$ be a vector space. Show that

1. if $a, b \in F$ and α is a non-zero element of V, then
$$a\alpha = b\alpha \Rightarrow a = b;$$
2. if $\alpha, \beta \in V$ and a is a non-zero element of F, then
$$a\alpha = a\beta \Rightarrow \alpha = \beta.$$ 2+2=4

(ii) Prove that if two vectors are linearly dependent, then one of them is a scalar multiple of the other. 3

(iii) Show that the intersection of any two sub-spaces W_1 and W_2 of a vector space $V(F)$ is also a subspace of $V(F)$. 3

(b) (i) Show that there exists a basis for each finite dimensional vector space. 4

(ii) Prove that the linear span $L(S)$ of any subset S of a vector space $V(F)$ is a subspace of V generated by S i.e., $L(S) = \{S\}$. 4

(iii) In $V_3(R)$, where R is the field of real numbers, then examine the linear dependence of the vectors $\{(2, 1, 2), (8, 4, 8)\}$. 2

(Continued)

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(a) If a, b, c are the roots of $x^3 + px^2 + qx + r$, then find the value of $\sum \frac{b^2 + c^2}{bc}$.

(b) Find the condition that the cubic equation $x^3 - px^2 + qx - r = 0$ should have roots in GP.

(c) If $a_1 x^2 + a_2 x + a_3 = 0$ and x and y be two roots of the equation, then find the values of $x + y$ and xy.

10. Answer any *one* of the following questions : 10

(a) (i) Solve the Cardan's method 3
$$x^3 + 6x^2 - 12x + 32 = 0$$

(ii) Prove that in an equation with real coefficients, imaginary roots occur in pairs. 4

(iii) Show that an algebraic equation of degree n has n roots. 3

(Turn Over)

(b) *(i)* Determine whether the statement is true or false :

$f(x) = (x-2)(x-7)$ has always three distinct real roots.

Also find the roots of the equation $x^3 + 7x^2 + 14x + 8 = 0$ if they are in AP.

1+4=

(ii) Show that every equation of odd degree has at least one real root. Also if the equation $ax^3 + 3bx^2 + 3cx + d = 0$ has two equal roots, then prove that $(bc-ad)^2 = 4(b^2 - ac)(c^2 - bd)$ and the equal root is $\frac{1}{2}\frac{(bc-ad)}{(ac-b^2)}$.

2+3=5

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