

**2025/FYUG/ODD/SEM/
MTMIDC-201T/071**

FYUG Odd Semester Exam., 2025

MATHEMATICS

(3rd Semester)

Course No. : MTMIDC-201T

(Basic Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

UNIT—I

1. Answer any *four* of the following questions :

1×4=4

- (a) Define continuity of a function.
- (b) Is a polynomial function always continuous?
- (c) Find the value of

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$$

(2)

(d) Write the value of $\lim_{x \rightarrow 0} \frac{\sin x}{x}$

(e) Find the value of $\lim_{x \rightarrow 0} \frac{\cos x}{\pi - x}$

2. Answer any one of the following questions :

(a) Evaluate :

$$\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 - 3x + 2}$$

(b) Evaluate :

$$\lim_{x \rightarrow 0} \frac{\sin 7x}{\sin 6x}$$

3. Answer any one of the following questions :

(a) (i) Evaluate :

$$\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\theta}$$

(ii) Evaluate :

$$\lim_{x \rightarrow 0} \frac{\sqrt{1-x} - 1}{x}$$

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(3)

(i) If

$$f(x) = \begin{cases} \cos x, & x \geq 0 \\ x + k, & x < 0 \end{cases}$$

and $\lim_{x \rightarrow 0} f(x)$ exists, find k.

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(ii) Evaluate :

4

$$\lim_{n \rightarrow \infty} \frac{1+2+3+\dots+n}{n^2+3}$$

UNIT—II

Answer any four of the following questions :

1×4=4

- (a) Write the value of $\frac{d}{dx}(\sqrt{x})$.
- (b) Write the value of $\frac{d}{dx}(\tan x)$.
- (c) Write the value of $\frac{d}{dx}(\log x)$.
- (d) Write the value of $\frac{d}{dx}(\cos 3x)$.
- (e) Write the value of $\frac{d}{dx}(x \cdot e^x)$.

3. Answer any one of the following questions : 2

- (a) Write the derivative of the function $f(x) = \frac{1}{x}$, by definition.

(Turn Over)

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(4)

(b) Evaluate :

$$\frac{d}{dx} (\sec x \cdot \tan x)$$

6. Answer any one of the following questions :

(a) (i) Evaluate :

$$\frac{d}{dx} \left(\frac{x + \sin x}{x + \cos x} \right)$$

(ii) If $y = x + \frac{1}{x}$, then prove that

$$x^2 \frac{dy}{dx} - xy + 2 = 0$$

(b) (i) Find the derivative of

$$\frac{x^n - a^n}{x - a}$$

for some constant a .

(ii) Find from the definition $f'(2)$ if $f(x) = \sqrt{x+1}$.

UNIT—III

7. Answer any four of the following questions :

(a) When is a function $f(x)$ said to be maximum at $x = c$?

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(5)

(b) Find the rate of change of the area of a circle w.r.t. its radius r , when $r = 6$ cm.

(c) Write the geometrical significance of $\frac{dy}{dx}$.

(d) Is $f(x) = x^3 - 6x^2 + 12x - 18$ an increasing function?

(e) Write down the minimum value of $f(x) = x^2$, $x \in \mathbb{R}$.

8. Answer any one of the following questions : 2

(a) Find the intervals on which the function $f(x) = -2x^3 - 9x^2 - 12x + 1$ is strictly increasing.

(b) The radius of a circle is increasing at the rate of 0.7 cm/s. What is the rate of increase of its circumference?

9. Answer any one of the following questions : 8

(a) (i) The surface area of a spherical balloon is increasing at $2 \text{ cm}^2/\text{sec}$. At what rate is the volume of the bubble increasing when the radius of the bubble is 6 cm?

(Turn Over)

(6)

- (ii) Prove that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$.
- (b) (i) Find the maximum and minimum values of $2x^3 - 24x + 107$ on the interval $[-3, 3]$.
- (ii) Find the intervals on which the function $f(x) = 2x^3 - 15x^2 + 36x + 6$ is (1) increasing and (2) decreasing.

UNIT—IV

10. Answer any four of the following questions : 1×4=4

- (a) Write the value of $\int x^3 dx$.
- (b) Write the value of $\int (ax+b)^3 dx$.
- (c) Write the value of $\int \frac{1}{x^2} dx$.
- (d) Write the value of $\int \sqrt[3]{x} dx$.
- (e) Write the value of $\int 5^x dx$.

11. Answer any one of the following questions :

(a) Evaluate : $\int \tan^2 x dx$

(b) Evaluate :

$$\int \left(x^2 + \frac{1}{x^2}\right)^3 dx$$

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(7)

Answer any one of the following questions :

(i) Evaluate : $\int \frac{1}{1+\tan x} dx$

(ii) Evaluate : $\int \frac{(1+\cos x)}{(1-\cos x)} dx$

(i) Evaluate : $\int \sqrt{\frac{1+x}{1-x}} dx$

(ii) Evaluate : $\int \frac{(x-1)}{(x+1)(x-2)} dx$

UNIT—V

Answer any four of the following questions : 1×4=4

(a) Evaluate $\int_1^3 x^4 dx$.

(b) Evaluate $\int_0^{16} x^{3/4} dx$.

(c) Evaluate $\int_1^4 \frac{dx}{\sqrt{x}}$.

(d) Evaluate $\int_0^{\pi/4} \tan^2 x dx$.

(Turn Over)

- (e) Write down the formula of area of a region bounded by the curve $y = f(x)$, x -axis and the ordinates $x = a$, $x = b$.

14. Answer any *one* of the following questions :

- (a) Prove that

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

- (b) If $f(x)$ is an even function, prove that

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$$

15. Answer any *one* of the following questions :

- (a) (i) Show that

$$\int_0^{\pi/2} \log(\tan x) dx = 0$$

- (ii) Prove that

$$\int_0^{\pi/2} \left(\frac{\sin x}{\sin x + \cos x} \right) dx = \frac{\pi}{4}$$

- (b) (i) Evaluate $\int_0^1 x(1-x)^n dx$.

- (ii) Prove that

$$\int_0^{\pi/4} \log(1 + \tan x) dx = \frac{\pi}{8} (\log 2)$$

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