2025/FYUG/ODD/SEM/
MTMDSC–202T/068

FYUG Odd Semester Exam., 2025

MATHEMATICS

(3rd Semester)

Course No. : MTMDSC–202T

(Ordinary Differential Equation)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

Answer any *two* of the following questions : 2×2=4

(a) Define order and degree of a differential equation, and give an example of each.

(b) Find the differential equation of the family of curves $y = Ae^{3x} + Be^{5x}$, for different values of A and B.

(c) Solve :

$$x\sqrt{1+y^2}\,dx + y\sqrt{1+x^2}\,dy = 0$$

swer any one of the following questions :

(a) Solve the following :

(i) $(2x+4y+3)\frac{dy}{dx}=(2y+x+1)$

(ii) $x\,dy-y\,dx=\sqrt{x^2+y^2}\,dx$

(iii) $(e^y+1)\cos x\,dx+e^y\sin x\,dy=0$

(b) Solve the following :

(i) $\frac{dy}{dx}=\frac{(y-x-1)}{(y+x+5)}$

(ii) $\left(\frac{dy}{dx}\right)\tan y=\sin(x+y)+\sin(x-y)$

(iii) $x\,dx+y\,dy=\frac{a^2(x\,dy-y\,dx)}{x^2+y^2}$

UNIT—II

3. Solve any two of the following :

(a) $x\,dx+y\,dy=m(x\,dy-y\,dx)$

(b) $x\cos x\left(\frac{dy}{dx}\right)+y(x\sin x+\cos x)=1$

(c) $(y-px)(p-1)=p$, where $p\equiv\frac{d}{dx}$

(Continue

answer any one of the following questions : 10

(a) Solve the following : 3+3+4=10

(i) $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$

(ii) $(1+y^2)+(x-e^{\tan^{-1}y})\left(\frac{dy}{dx}\right)=0$

(iii) $x^2(y-px)=yp^2$, where $p\equiv\frac{d}{dx}$

(b) (i) Solve : 3

$y=2px+p^4x^2$

where $p\equiv\frac{d}{dx}$

(ii) Find the differential equation of the family of circles

$x^2+y^2+2cx+2c^2-1=0$

(c is an arbitrary constant).

Determine singular solution of the differential equation. 2+2=4

(iii) Solve : 3

$\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$

(Turn Over)

UNIT—III

5. Answer any *two* of the following questions : 2×2

(a) What is a non-homogeneous linear differential equation of order greater than one? Give an example of a non-homogeneous linear differential equation of order 2.

(b) When are functions said to be linearly independent? Are $y_1(x) = e^x$ and $y_2(x) = e^{2x}$ linearly independent?

(c) Solve :

$$(D^4 + 2D^3 + 3D^2 + 2D + 1)y = 0$$

6. Answer any *one* of the following questions :

(a) *(i)* Solve :

$$(D^2 + a^2)y = \sec ax$$

(ii) Solve :

$$(D^2 - 2D + 4)y = e^x \cos x$$

(iii) Prove that the Wronskian of two solutions of the differential equation

$$y_2 + py_1 + Qy = 0$$

where P, Q are either constants or functions of x alone, is either identically zero or never zero.

Continued

(b) *(i)* If $y_1(x)$ and $y_2(x)$ are any two linearly independent solutions of the homogeneous differential equation $y_2 + Py_1 + Qy = 0$ and y_0 is any particular solution of the non-homogeneous differential equation $y_2 + Py_1 + Qy = R$, then show that the general solution of the non-homogeneous equation is $y_0 + c_1y_1 + c_2y_2$, where c_1 and c_2 are arbitrary constants. 4

(ii) Solve : 3

$$(D^2 - 2D + 1)y = xe^x \sin x$$

(iii) Solve : 3

$$(D^3 + 2D^2 + D)y = e^{2x} + x^2 + x$$

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Write the general solution of non-homogeneous differential equation using variation of parameters.

(b) Solve :

$$(x^2D^2 - 4xD + 6)y = x$$

(c) Solve :

$$(D^2 - 3D + 2)y = e^x$$

(Turn Over)

Answer any one of the following questions :

(a) (i) Solve : $(x^3D^3 + 2x^2D^2 + 2)y = 10\left(x + \frac{1}{x}\right)$ 10

(ii) Apply the method of variation of parameters to solve $y_2 - y = \frac{2}{1+e^x}$. 5

(b) (i) Find the orthogonal trajectories of the family of semi-cubical parabolas $ay^2 = x^3$, where a is a parameter. 5

(ii) Solve

$$\frac{d^2y}{dx^2} - 9y = x + e^{2x} - \sin 2x$$

using method of undetermined coefficients for finding particular integral. 5

UNIT—V

Answer any *two* of the following questions : 2×2=4

(a) What is a total differential equation? Write the condition for exactness of a total differential equation.

(continued)

(b) Show that $(2x + y^2 + 2xz)\,dx + 2xy\,dy + x^2\,dz = 0$ is integrable.

(c) Solve : $\frac{dx}{x^2 + 2y^2} = \frac{dy}{-xy} = \frac{dz}{xz}$

Answer any *one* of the following questions : 10

(a) Solve the following : 5+5=10

(i) $\frac{dx}{x(x^2+3y^2)} = \frac{dy}{-y(3x^2+y^2)} = \frac{dz}{2z(y^2-x^2)}$

(ii) $(x^2y - y^3 - y^2z)\,dx + (xy^2 - x^2z - x^3)\,dy + (xy^2 + x^2y)\,dz = 0$

(b) Solve the following : 5+5=10

(i) $\frac{dx}{dt} + 2x - 3y = t$

$\frac{dy}{dt} - 3x + 2y = e^{2t}$

(ii) $\frac{dx}{mz - ny} = \frac{dy}{nx - lz} = \frac{dz}{ly - mx}$

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