

**2025/FYUG/ODD/SEM/
STADSC-102T/034**

FYUG Odd Semester Exam., 2025

STATISTICS

(1st Semester)

Course No. : STADSC-102T

(Calculus)

Full Marks : 70

Pass Marks : 28

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

UNIT—I

1. Answer any *two* of the following questions :

2×2=4

(a) Define limit and continuity.

(b) Show that

$$\frac{d}{dx} \sin x = \cos x$$

(c) Evaluate :

$$\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$$

(2)

2. Answer any one of the following questions

(a) (i) State and prove Leibnitz's theorem

(ii) If $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$, then show that

(1) $(1-x^2)y_2 - 3xy_1 - y = 0$

(2) $(1-x^2)y_{n+2} - (2n+3)xy_{n+1} - (n+1)^2 y_{n+1} = 0$

(b) (i) Show that $\lim_{x \rightarrow 0} \sin \frac{1}{x}$ does not exist

(ii) Prove that differentiable functions are continuous but the converse may not be true.

(iii) Find f_{xx} , f_{yy} , f_{xy} , f_{yx} for the function

$f(x, y) = x^3 + 3x^2y + 3xy^2 + y^3$

UNIT—II

3. Answer any two of the following questions:

(a) Define Jacobian.

(b) Define singular point.

(c) Define convexity and concavity.

4. Answer any one of the following questions:

(a) (i) Define maxima and minima of a function at a point $x = c$.

(3)

2. Answer any one of the following questions

(a) Find for what values of x the following function is maximum and minimum respectively :

$f(x) = 2x^3 - 21x^2 + 36x - 20$

(ii) Find all maxima and minima of the function

$4x^2 - xy + 4y^2 + x^3y - 4$

(b) (i) If $u = \frac{x^2 + y^2 + z^2}{x}$, $v = \frac{x^2 + y^2 + z^2}{y}$, $w = \frac{x^2 + y^2 + z^2}{z}$

find the Jacobian of $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

(ii) Show that the curve $y = x^3$ has point of inflexion at $x = 0$.

(iii) Examine if $x^{\frac{1}{x}}$ possess a maximum or minimum.

UNIT—III

5. Answer any two of the following questions :

(a) Evaluate :

$\int \frac{1}{2\sqrt{x}} dx$

(4)

(b) What is the geometric interpretation of definite integral?

(c) Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

6. Answer any one of the following questions :

(a) (i) Show that

$$\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cdot \cos x} dx = 0$$

(ii) Evaluate $\iint_R \sqrt{x^2 + y^2} dx dy$, the field of integration R being R , the origin in XY plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

(b) (i) Prove that

$$\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$$

(ii) Show that

$$\sqrt{\pi} = 2^{2m-1} \cdot \frac{\Gamma(m)\Gamma\left(m+\frac{1}{2}\right)}{\Gamma(2m)}$$

(5)

UNIT—IV
following questions : $2 \times 2 = 4$

1. Answer any two of the following questions with differential equation with

Define a

example.

differential equation?

Define order and degree of differential equation.

equation.

(b) What is an exact differential equation?

10

(c) Answer any one of the following questions :

For a given

(i) $y = A \cos(x+B)$, where A, B are constants. Formulate differential equation.

3

(ii) Solve :

$x \sqrt{1-y^2} dx + y \sqrt{1-x^2} dy = 0$

3

(iii) Solve :

$\frac{dy}{dx} = \frac{x^2 y}{x^3 + y^3}$

4

(b) (i) For a given equation

$y = e^{-x} (A \cos x + B \sin x)$

3

where A and B are constants, formulate differential equation.

(Turn Over)

(ii) Solve :

$$(y^2 e^x + 2xy) dx - x^2 dy = 0$$

(7)

(iii) Solve :

$$\frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + 4y = 0$$

(b)

(i) Solve :

$$(D^2 - 2D + 1)y = x^2 \cdot e^{3x}$$

(ii) Solve :

$$\frac{d^3 y}{dx^3} + 6 \frac{d^2 y}{dx^2} + 11 \frac{dy}{dx} + 6y = 0$$

UNIT—V

9. Answer any two of the following questions :

(a) Define homogeneous and linear differential equation.

(b) State degree and order of partial differential equation.

(c) State the conditions for real and distinct, repeated and complex roots of the auxiliary equation.

10.

Answer any one of the following questions :

(a) (i) Form a partial differential equation from $x^2 + y^2 + (z - c)^2 = a^2$ by eliminating a, c .

(ii) Solve :

$$(D^2 + D)y = \cos 2x$$

