FYUG Odd Semester Exam., 2025

PHYSICS

(1st Semester)

Course No. : PHYDSC-101T

(Mathematical Physics—I)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) Define Hermitian and skew-Hermitian matrices.

(b) Find the value of λ, if $\vec{A} = \hat{i} + \lambda\hat{j} + 5\hat{k}$ is perpendicular to $\vec{B} = 2\hat{i} + \hat{j} - \hat{k}$.

(c) State the physical interpretation of vector product.

2. (a) What are symmetric and skew-symmetric matrices? Show that every square matrix can be expressed as the sum of symmetric and skew-symmetric matrices. 2+

(b) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & -6 & -7 \end{bmatrix}$$

OR

3. (a) Find the eigenvalues and eigenvector of the matrix

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

(b) (i) Show that

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \vec{B}\cdot(\vec{C}\times\vec{A}) = \vec{C}\cdot(\vec{A}\times\vec{B})$$

(ii) Prove that

$$\vec{A}\times(\vec{B}\times\vec{C}) + \vec{B}\times(\vec{C}\times\vec{A}) + \vec{C}\times(\vec{A}\times\vec{B}) = 0$$

UNIT—II

4. Answer any *two* of the following questions : 2×

(a) What are meant by order and degree of a differential equation? Give example.

(b) Using Wronskian, show that the functions e^{ax} and e^{-ax} are linearly independent.

(c) What is linear differential equation? Write the general form of first-order linear differential equation.

5. (a) Solve : 5

$$\frac{dy}{dx} + y = x$$

(b) Using the method of separation of variables, solve the following differential equation : 5

$$\frac{dy}{dx} = \frac{y+1}{x+1}$$

Given $y = 1$ at $x = 0$.

OR

6. (a) Discuss the method to find the complementary function of second-order linear homogeneous equation with constant coefficients. 6

(b) Solve the following differential equation : 4

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$$

UNIT—III

7. Answer any *two* of the following questions : 2×2

(a) Show that the vector

$$\vec{F} = (x+3y)\hat{i} + (y-3z)\hat{j} + (x-2z)\hat{k}$$

is solenoidal.

(b) Show that curl grad $\phi = 0$.

(c) What is Stokes' theorem?

8. (a) Define divergence of a vector function. Explain the physical significance of divergence. 2+3

(b) Find the directional derivative of the function $xy^2 + yz^3$ at the point (1, −1, 1) in the direction of (3, 1, −1).

(c) What is Laplacian?

OR

9. (a) State and prove Gauss divergence theorem.

(b) Evaluate grad $(\frac{1}{r})$, where r is a position vector.

UNIT—IV

10. Answer any *two* of the following questions : 2×2=4

(a) What are curvilinear coordinates? When are they orthogonal?

(b) Express elements of volume in orthogonal curvilinear coordinates.

(c) Prove that $\hat{e}_1 = h_2 h_3 (\vec{\nabla} u_2 \times \vec{\nabla} u_3)$ where the symbols have their usual meanings.

11. (a) Find the expressions for divergence and curl of a vector in orthogonal curvilinear coordinates. 3+3=6

(b) Show that cylindrical coordinate system is orthogonal. 4

OR

12. Explain the spherical polar coordinate system. Find the expressions for length and volume elements in spherical polar coordinates. Find the expression for gradient and divergence in this system of coordinates. 3+3+4=10

UNIT—V

13. Answer any *two* of the following questions : 2×2=4

(a) Find the value of $\Gamma(\frac{3}{2})$.

(b) Find the recursion relation of Γ-function.

(c) State Simpson's $\frac{1}{3}$rd rule.

14. (a) Find the relation between beta and gamma function. 4

(b) Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$. 3

(c) Show that

$$\int_0^1 \frac{y^{m-1}+y^{n-1}}{(1+y)^{m+n}}\,dy = \frac{\Gamma(m)\Gamma(n)}{\Gamma(mn)}$$

3

OR

15. (a) Explain the method of finding solution of algebraic equation by bi-section method. Find the real root of the equation $x^3 - x - 1 = 0$. 3+4=7

(b) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\, d\theta$ by using Γ-function. 3

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