2025/FYUG/ODD/SEM/
MTMDSM–101T/064

FYUG Odd Semester Exam., 2025

MATHEMATICS

(1st Semester)

Course No. : MTMDSM–101T

(Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

. Answer any *two* of the following questions : $2 \times 2 = 4$

(a) Write the ε-δ definition of limit.

(b) Examine the continuity of $f(x)$ at $x = 0$, where $f(x) = |x|$.

(c) Find y_n if $y = \cos 3x$.

2. Answer *either* [(*a*) and (*b*)] or [(*c*) and (*d*)]

(*a*) (*i*) Evaluate

$$\underset{x\to\infty}{\mathrm{Lt}}\ \frac{x^2-5x+6}{2x^2+7x-9}$$

(*ii*) Use ε-δ definition of limit to show that $\underset{x\to 0}{\mathrm{Lt}}\ x^2 \sin\frac{1}{x}=0$.

(*b*) Find $\frac{dy}{dx}$ if

$$y=\tan^{-1}\left\{\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}\right\}$$

(*c*) If
$$f(x)=3+2x \quad \text{for } -\frac{3}{2}\le x<0$$
$$=3-2x \quad \text{for } 0\le x<\frac{3}{2}$$
$$=-3-2x \quad \text{for } x\ge\frac{3}{2}$$

show that $f(x)$ is continuous at $x=0$ but discontinuous at $x=\frac{3}{2}$.

(*d*) State and prove Leibnitz's theorem. 1

UNIT—II

3. Answer any *two* of the following questions : 2

(*a*) State Lagrange's mean value theorem.

(Continued)

(*b*) Write the geometrical meaning of Rolle's theorem.

(*c*) What is the maximean value of $-x^2+4x+5$?

4. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] : 10

(*a*) (*i*) Evaluate

$$\underset{x\to 0}{\mathrm{Lt}}\ \frac{e^x-e^{-x}-2x}{x-\sin x}$$ 2

(*ii*) Evaluate

$$\underset{x\to 0}{\mathrm{Lt}}\ \frac{x-\tan x}{x^3}$$ 3

(*b*) Show that the relative maximum value of $x+\frac{1}{x}$ is less than its relative minimum value. 5

(*c*) Write Taylor's series. Expand $\sin x$ using Maclaurin's infinite series. 1+4=5

(*d*) Evaluate

$$\underset{x\to 0}{\mathrm{Lt}}\ (\cot^2 x)^{\sin x}$$ 5

(Turn Over)

UNIT—III

5. Answer any *two* of the following questions :

(a) Write the statement of Euler's theorem on homogenous functions (two variables).

(b) Find f_x and f_y where $f(x, y) = ax^2 + 2hxy + by^2$.

(c) Show that in any curve

$$\frac{\text{subnormal}}{\text{subtangent}} = \left(\frac{\text{length of normal}}{\text{length of tangent}}\right)^2$$

6. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) If $u = e^{xyz}$, then prove that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$$

(b) (i) If $V = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, then show that $xV_x + yV_y = \tan V$.

(ii) Show that the subnormal at any point of a parabola is of constant length.

(c) If $u = f\left(\frac{y}{x}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

(d) (i) Find the equation of the normal to the curve $y = x^3 - 2x^2 - x + 5$ at the point (0, 5). 3

(ii) Find the equation of the tangent to the curve $y(x-2)(x+3) - x + 7 = 0$ at the point where it cuts the *X*-axis. 4

UNIT—IV

7. Answer any *two* of the following questions : 2×2=4

(a) Given

$$f(x) = x^2, \quad \text{for } 0 \le x < 1$$
$$= \sqrt{x}, \quad \text{for } 1 \le x \le 2$$

evaluate $\int_0^2 f(x)\,dx$.

(b) Prove that

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

(c) Prove that

$$\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$$

8. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Obtain the reduction formula for $\int \tan^n x\,dx$. 5

(b) Prove that

$$\int_0^{\pi/2}(a\cos^2 x + b\sin^2 x)\,dx = \frac{\pi}{4}(a+b)$$

(c) Show that

$$\int_0^{\pi}\log(\sin x)\,dx = \frac{\pi}{2}\log\frac{1}{2}$$

(d) Prove that

$$\int_0^{\pi/2}\cos^n x\,dx = \begin{cases}\frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} & \text{if } n \text{ is even} \\ \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{4}{5}\cdot\frac{2}{3}\cdot 1 & \text{if } n \text{ is odd}\end{cases}$$

UNIT—V

9. Answer any *two* of the following questions : 2×2

(a) Write the formula for finding length of the curve $y = f(x)$ from $x = a$ to $x = b$.

(b) What is the surface area of the solid generated by revolving a semicircular arc of radius C about its diameter?

(c) Write the formula for calculating the volume of the solid generated by revolving a curve about y-axis.

10. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Find the area included between the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ and its base. 5

(b) Find the area of the surface of the solid generated by revolving the arc of the parabola $y^2 = 4ax$, bounded by the latus rectum about x-axis. 5

(c) Find the length of the perimeter of the circle $x^2 + y^2 = a^2$. 5

(d) Find the volume of the ellipsoid generated by the revolution of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

about the major axis. 5

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