2025/FYUG/ODD/SEM/
MTMDSC–102T/063

FYUG Odd Semester Exam., 2025

MATHEMATICS

(1st Semester)

Course No. : MTMDSC–102T

(Differential Calculus)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* of the following questions : 2×2=4

(a) Find

$$\lim_{x \to 0} x^2 \sin\frac{1}{x}$$

(b) Define continuity. Give example.

(c) Using first principle, find $\frac{dy}{dx}$ where $y = \sin x$.

2. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] :

(*a*) Prove that

$$\lim_{x \to a} f(x) \cdot g(x) = lm \text{ when } \lim_{x \to a} f(x) = l \text{ an[d]} \lim_{x \to a} g(x) = m$$

Hence, find the limit of $\sin x \tan x$ a[t] $x = \frac{\pi}{4}$. 4

(*b*) Discuss the types of discontinuity of [a] function.

(*c*) Show that $\sin x + \cos x$ is continuous o[n] the set of real numbers.

(*d*) Define derivative of a function. Prov[e] that if a function is differentiable at [a] point then it is continuous at that point. 1

UNIT—II

3. Answer any *two* of the following questions : 2

(*a*) If $y = e^{ax}$, then find y_n.

(*b*) An edge of a variable cube is increasing at the rate of 3 cm/s. How fast is the volume of the cube increasing?

(*c*) If $y = \sin x$, find the value of $\frac{d^{15}y}{dx^{15}}$.

4. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] : 10

(*a*) If $y = e^{ax} \sin bx$, find y_n. 5

(*b*) If $y = e^{m \sin^{-1} x}$, then show that

$$(1 - x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 + m^2) y_n = 0$$

Also find $y_n(0)$. 5

(*c*) If $y = \frac{1}{x^2 + a^2}$, then find y_n. 5

(*d*) If $x \sin\theta + y \cos\theta = a$ and $x \cos\theta - y \sin\theta = b$, then prove that

$$\frac{d^p x}{d\theta^p} \cdot \frac{d^q y}{d\theta^q} - \frac{d^q x}{d\theta^q} \cdot \frac{d^p y}{d\theta^p}$$

is a constant. 5

UNIT—III

5. Answer any *two* of the following questions : 2×2=4

(*a*) Verify Rolle's theorem for the function $f(x) = x^3 - 4x$ for all $x \in [-2, 2]$.

(*b*) State Taylor's theorem.

(*c*) Describe the geometrical interpretation of Lagrange's mean value theorem.

6. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] :

(*a*) State and prove Cauchy's mean value theorem.

(*b*) Show that

$$\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2},$$

if $0 < u < v$.

(*c*) Define Maclaurin's infinite series. Hence, express $\cos x$ in terms of infinite series.

(*d*) Prove that a conical tent of given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base.

UNIT—IV

7. Answer any *two* of the following questions :

(*a*) Find the radius of curvature of $s = 4a \sin \Psi$.

(*b*) Define tangent and normal of a curve.

(*c*) Find $\frac{ds}{d\theta}$, where $r = a(1 + \cos\theta)$.

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8. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] : 10

(*a*) Find the equation of tangent and normal at (1, –1) to the curve

$$x^3 + xy^2 - 3x^2 + 4x + 5y + 2 = 0.$$ 5

(*b*) Find the radius of curvature at the origin (0, 0) of the curve

$$3x^2 + xy + y^2 - 4x = 0$$ 5

(*c*) Show that curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ cuts each other orthogonally. 5

(*d*) Define subtangent, subnormal, length of normal and length of tangent. Hence prove

$$\frac{\text{subnormal}}{\text{subtangent}} = \left(\frac{\text{length of normal}}{\text{length of tangent}}\right)^2$$ 5

UNIT—V

9. Answer any *two* of the following questions : 2×2=4

(*a*) Find f_x and f_y, when

$$f(x, y) = \tan^{-1}(y / x)$$

(*b*) If $u = f\left(\frac{y}{x}\right)$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.$$

(Turn Over)

(c) Prove that the curve $y = e^x$ is convex the x-axis at every point.

10. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Find the asymptotes of the curve

$$x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$$

(b) If $f(x, y, z)$ has all the partial derivative continuous such that

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = nf(x, y, z)$$

where n is a positive integer. Prove tha $f(x, y, z)$ is a homogeneous function degree n.

(c) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, then show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

(d) Find the asymptotes of the following : 2½

(i) $x^2 - 4y^2 = 1$

(ii) $(r - a)\sin\theta = b$

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