2025/FYUG/ODD/SEM/
MTMDSC–101T/062

FYUG Odd Semester Exam., 2025

MATHEMATICS

(1st Semester)

Course No. : MTMDSC–101T

(Higher Algebra and Trigonometry)

Full Marks : 70
Pass Marks : 28

Time : 3 hours

The figures in the margin indicate full marks for the questions

UNIT—I

1. Answer any *two* questions from the following : $2\times2=4$

(a) Find all the values of $1^{1/3}$.

(b) Express

$$\left(\frac{\cos\theta + i\sin\theta}{\sin\theta + i\cos\theta}\right)^5$$

in the form $x+iy$.

(c) Expand $\cos^2\theta$ in powers of θ.

2. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] :

(*a*) (*i*) If

$$x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$$

then show that $x_1 \cdot x_2 \cdot x_3 \cdots \infty = -1$

(*ii*) Prove that the nth roots of unity form a series in GP.

(*b*) Prove that

$$\frac{\sin^3\theta}{3!} = \frac{\theta^3}{3!} - (1+3^2)\frac{\theta^5}{5!} + (1+3^2+3^4)\frac{\theta^7}{7!} - \cdots$$

(*c*) If $a = \cos\theta_1 + i\sin\theta_1$, $b = \cos\theta_2 + i\sin\theta_2$, $c = \cos\theta_3 + i\sin\theta_3$ and $a+b+c = abc$, then show that

$$\cos(\theta_2 - \theta_3) + \cos(\theta_3 - \theta_1) + \cos(\theta_1 - \theta_2) + 1 = 0$$

(*d*) Prove that

$$\sin m\theta = \sec^m\theta\left\{m\tan\theta - \frac{m(m+1)(m+2)}{3!}\tan^3\theta + \cdots\right.$$

when $\tan\theta < 1$.

UNIT—II

3. Answer any *two* questions from the following : 2×2=4

(*a*) Prove that

$$x^i = e^{-2n\pi}\{\cos(\log x) + i\sin(\log x)\}$$

(*b*) Write the exponential values of $\sin x$ and $\cos x$.

(*c*) Write the expansions of $\sinh x$ and $\cosh x$.

4. Answer *either* [(*a*) and (*b*)] *or* [(*c*) and (*d*)] : 10

(*a*) Prove that i^i is real. Hence show that the values of i^i can be arranged so that they form a GP. 3+2=5

(*b*) Prove that

$$\log\cos\theta = -\log 2 + \cos 2\theta - \frac{1}{2}\cos 4\theta + \frac{1}{3}\cos 6\theta - \cdots \quad 5$$

(*c*) If $\tan x = n\tan y$, then find a series for x. 4

(*d*) (*i*) If $\cos^{-1}(\alpha + i\beta) = \theta + i\phi$, then show that $\alpha^2\operatorname{sech}^2\phi + \beta^2\operatorname{cosech}^2\phi = 1$. 3

(*ii*) If $u = \log\tan(\frac{\pi}{4} + \frac{\theta}{2})$, then prove that

$$\tanh\frac{u}{2} = \tan\frac{\theta}{2}. \quad 3$$

(Turn Over)

UNIT—III

5. Answer any *two* questions from following :

(a) Give an example of a relation which symmetric but neither reflexive transitive. Justify your answer.

(b) Using truth table, show th $(\sim p \wedge q) \wedge (p \vee \sim q)$ is a contradiction.

(c) (i) Let p be the statement "Ravi i rich." and q be the statement "Rav is happy." Write the following statement in symbolic form :

"Ravi is poor or he is both rich and unhappy."

(ii) Write the negation of the following statement :

"No nice people are dangerous."

6. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) Prove that every partition on a non-empty set determines an equivalence relation on the set.

(b) (i) Prove the logical equivalence

$$\sim (p \wedge q) \equiv (\sim p) \vee (\sim q)$$

(ii) Write the converse and contra-positive of the following statement :

"If it rains today, I will go to college tomorrow." 1+1=2

(c) Define equivalence relation on a set A. Let A be the set of all triangles in a plane and the relation R is defined as "is similar to". Then show that R is an equivalence relation in A. 2+3=5

(d) (i) Construct the truth table for the following compound statement :

$$(p \wedge q) \rightarrow (p \vee q)$$ 3

(ii) Rewrite the following statements with universal and existential quantifiers :

1. For any real x, 2^x is never negative.
2. There is no largest integer. 1+1=2

UNIT—IV

7. Answer any *two* questions from the following : 2×2=4

(a) If α, β, γ are the roots of the equation $x^3 + px + q = 0$, then find the value of $\Sigma\alpha^3$.

(b) Show that the equation $2x^7 - x^4 + 4x^3 - 5 = 0$ has at least four imaginary roots.

(c) Show that

$$n^n > 1.3.5.\cdots(2n-1)$$

8. Answer *either* [(a) and (b)] *or* [(c) and (d)] :

(a) (i) Find the condition that the roots of $x^3 + px^2 + qx + r = 0$ may be in AP.

(ii) Solve the following equation by Cardan's method :

$$x^3 - 3x + 1 = 0$$

(b) If $x, y, z \in R^+$ and $yz + zx + xy = 12$, then find the maximum value of xyz.

(c) State and prove Cauchy-Schwarz inequality.

(d) (i) Find the condition that the equation $x^3 + px^2 + qx + r = 0$ may have two roots equal in magnitude but opposite in sign.

(ii) Find the nature of the roots of the equation $x^3 + x^2 - 16x + 20 = 0$.

(Continued)

UNIT—V

9. Answer any *two* questions from the following : 2×2=4

(a) Define echelon form of a matrix.

(b) Show that no skew-symmetric matrix can be of rank 1.

(c) Prove that every singleton set containing non-zero vector is LI.

10. Answer *either* [(a) and (b)] *or* [(c) and (d)] : 10

(a) Prove that the rank of a product of two matrices cannot exceed the rank of either matrix. 5

(b) Solve the following equations by Gaussian elimination method : 5

$$2x + 5y + 3z = 9$$
$$3x + y + 2z = 3$$
$$x + 2y - z = 6$$

(Turn Over)

(c) Reduce the matrix

$$\begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -4 \\ 6 & -3 & 0 & -7 \end{pmatrix}$$

into echelon form and hence find th
rank of it.
1

(d) Define linearly independent and linearl
dependent sets of vectors. Examine th
vectors $(1, 2, 1), (3, -4, 7), (3, 1, 5)$ ar
linearly independent or linearl
dependent.
1+1

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