~~Orthogonal~~ Ordinary differential eqn

ODE (~~orthogonal~~ ordinary differential eqn) ÷ A differential eqn involving derivatives with respect to a single independent variable is called an ordinary differential eqn.

Ex. 1. $dy = (x + \sin x)\, dx$

Ex. 2. $\dfrac{d^3y}{dx^3} + \dfrac{dy}{dx} + y = e^x$

Partial diffential eqn: A differential eqn involving partial derivative with respect to more than one independent variable is called PDE.

Ex. $\dfrac{\delta^2 u}{\delta x^2} + \dfrac{\delta^2 u}{\delta y^2} + \dfrac{\delta^2 u}{\delta z^2} = 0$

Order of differential eqn ÷ The order of the highest order derivative in a differential eqn is called the order of the differential eqn.

Ex. $\dfrac{d^3y}{dx^3} + \left(\dfrac{dy}{dx}\right)^2 = e^x$ ∴ The order is 3

Ex. $\left(\dfrac{dy}{dx}\right)^3 + y = e^{2x}$ ∴ The order is 1.

Degree of differential eqn! The degree of a differential of a differential eqn is the degree of the highest derivative is called degree.

of diff eqn

Ex.1 $\frac{d^3y}{dx^3}+\left(\frac{dy}{dx}\right)^2=e^x$ The degree is 2

Ex.2 $\left(\frac{dy}{dx}\right)^3+y=e^{2x}$ The degree is 3.

Result: A diff. eqn contain following types of term $\log\left(\frac{dy}{dx}\right)$, $e^{\frac{dy}{dx}}$, $\sin\left(\frac{dy}{dx}\right)$, $\cos\left(\frac{dy}{dx}\right)$ then degree of this eqn's does not exist.

⟹ The general form of a first order orthogonal differential eqn is given by:

$$Mdx+Ndy=0$$

Separation of Variable:

Q. Solve $\frac{dy}{dx}=e^{x-y}+x^2e^{-y}$

Sol $\frac{dy}{dx}=e^x\cdot e^{-y}+x^2e^{-y}$

$\Rightarrow \frac{dy}{dx}=e^{-y}(e^x+x^2)$

$\Rightarrow \frac{dy}{e^{-y}}=(e^x+x^2)dx$

$\Rightarrow \int e^y dy=\int(e^x+x^2)dx$

$\Rightarrow e^y=e^x+\frac{x^3}{3}+c$

Q. Solve $e^x\tan y\,dx+(1-e^x)\sec^2y\,dy=0$

Sol $e^x\tan y\,dx=-(1-e^x)\sec^2y\,dy$

$\Rightarrow \frac{e^x}{1-e^x}\tan y\,dx=-\sec^2y\,dy$

$\Rightarrow \frac{e^x}{1-e^x}dx=-\frac{\sec^2x}{\tan y}dy$

$\Rightarrow \frac{e^x}{1-e^x}dx+\frac{\sec^2x}{\tan y}dy=0\,dx$

$\Rightarrow$ let, $1-e^x=t$ $\Rightarrow \int\frac{e^x}{1-e^x}dx+\int\frac{\sec^2x}{\tan y}dy=\int 0\,dx$

$\Rightarrow -e^xdx=dt$

$\Rightarrow e^xdx=-dt \Rightarrow -\int\frac{dt}{t}+\int\frac{\sec^2x}{\tan y}dy=\int 0\,dx$

$\Rightarrow -\log(1-e^x)+\log(\tan y)=\log c$

$\Rightarrow \log\frac{\tan y}{1-e^x}=\log c$

$\Rightarrow \frac{\tan y}{1-e^x}=c$

$\Rightarrow \tan y=c(1-e^x)$

Exact equation ÷ $M\,dx + N\,dy = 0$

$$\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$$

formula :- $\int M\,dx + \int \left(\text{terms of } N \text{ not containing } x\right) dy = c$

Q. Solve $(12x + 5y - 9)\,dx + (5x - 2y - 4)\,dy = 0$

Sol^n Given

$(12x + 5y - 9)\,dx + (5x - 2y - 4)\,dy = 0$

Compare it with $M\,dx + N\,dy = 0$

$\therefore M = 12x + 5y - 9 \qquad N = 5x - 2y - 4$

$\therefore \frac{\delta M}{\delta y} = 0 + 5 - 0 \qquad \frac{\delta N}{\delta x} = 5 - 0 - 0 = 5$

$= 5.$

Since, $\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x} = 5.$

$\therefore$ The given differential eq^n is exact.

$\therefore$ The sol^n of given Diff eq^n is given by.

$\int M\,dx + \int (\text{terms of } N \text{ not containing } x)\,dy = c$

$\Rightarrow \int (12x + 5y - 9)\,dx + \int (-2y - 4)\,dy = c$

$\Rightarrow \frac{\overset{6}{\cancel{12}}x^2}{\cancel{2}} + 5xy - 9x - \cancel{2}\frac{y^2}{\cancel{2}} - 4y = c$

$\Rightarrow x^2 + 5xy - 9x - y^2 - 4x = c$

In exact equation / Non-exact ÷

An eq^n which is of the form of $M\,dx + N\,dy = 0$ is said to be a non exact D.E. If it does not satisfies the condition $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\therefore \boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$ Non-exact diff. eq^n.

Integrating factor ÷ To make a exact differential equation into an exact differential equal we multiply with a suitable function $u(x,y) \neq 0$ then $u(x,y)$ is called an Integrating factor of $M\,dx + N\,dy = 0$

Linear differential equation : A differential eqn is called linear if (i) every dependent variable and every derivative occurs in first degree

(ii) No Product of dependent variable and derivative.

Ex. $dy = (x + \sin x)dx$

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \sin x$$

Q. Solve the differential eqn $(x^2+y^2)dx + 2xydy = 0$

Given, $(x^2+y^2)dx + 2xy\,dy = 0$

compare it with $Mdx + Ndy = 0$

$\therefore M = x^2 + y^2 \qquad N = 2xy$

$\Rightarrow \frac{\partial M}{\partial y} = 2y \qquad \frac{\partial N}{\partial x} = 2y$

Since, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\therefore$ The given eqn is exact.

$\therefore$ The soln of given diff eqn is

$\int M d\cancel{x} + \int (\text{term of N not containing x term})\, dy = c$

$\Rightarrow \int (x^2+y^2)\, dx + \int 0\, dy = c$

$\Rightarrow \frac{x}{3} + y^2 x = c$

Q. Solve the differential eqn $\frac{dy}{dx} + 2xy = 2e^{-x^2}$

Sol

$$\frac{dy}{dx} + 2xy = 2e^{-x^2}$$

$$\Rightarrow \frac{dy}{dx e^{-x^2}} + \frac{2xy}{e^{-x^2}} = 2$$

$$\Rightarrow e^{x^2}\frac{dy}{dx} + e^{x^2}2xy = 2$$

$$\Rightarrow \frac{d}{dx}\left[e^{x^2}y\right] = 2$$

$\Rightarrow$ Integrating both side $\boxed{\frac{d}{dx}e^{x^2}y = e^{x^2}\frac{dy}{dx} + y e^{x^2}2x}$

$$\int \frac{d}{dx}\left[e^{x^2}y\right] = \int 2dx$$

$$\Rightarrow e^{x^2}y = 2x + c$$

$$\Rightarrow y = \frac{2x}{e^{x^2}} + \frac{c}{e^{x^2}}$$